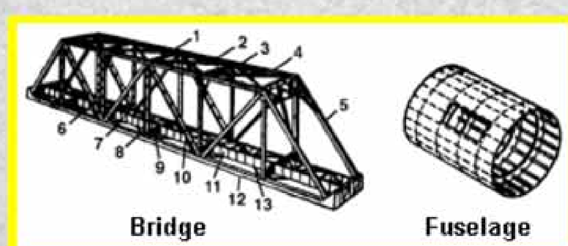
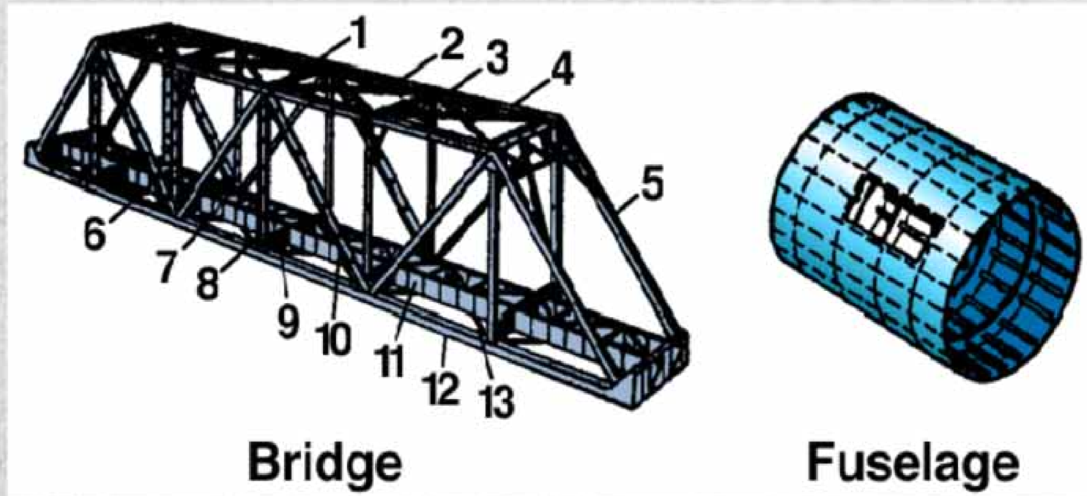
Definitions

Mechanics of Materials

Deals with

Art of idealizing actual solids and structures and their environments





Definitions

Mechanics of Materials

Deals with

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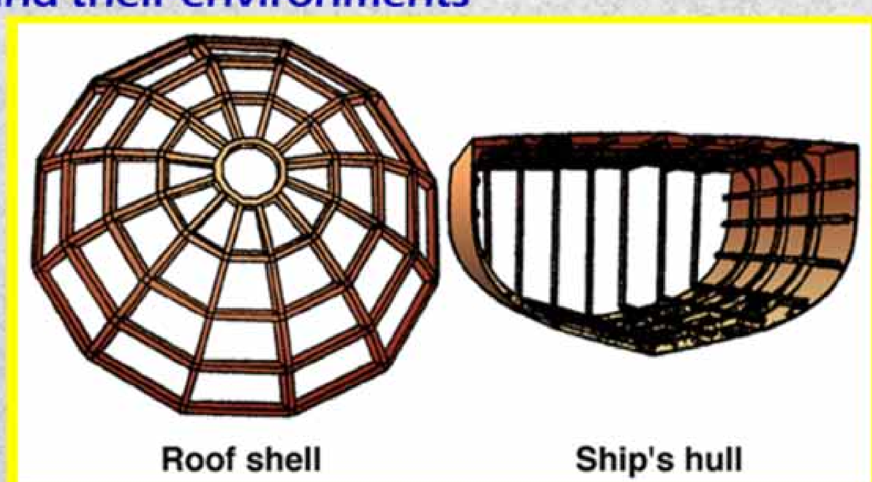
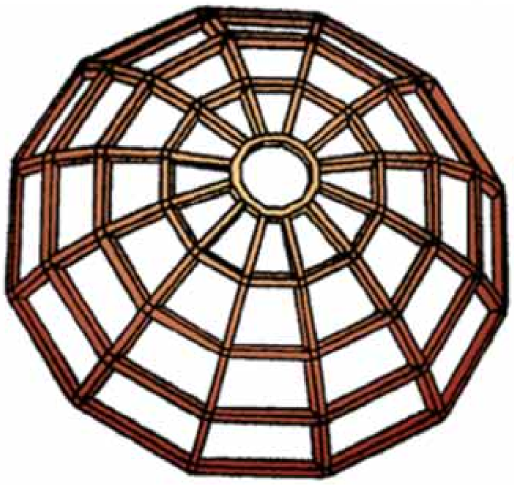
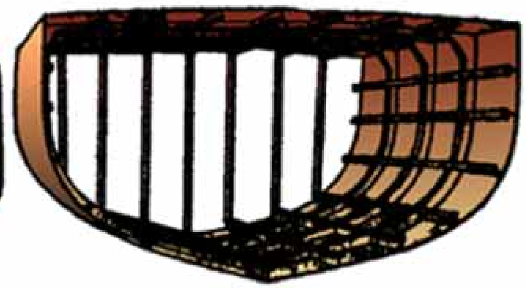



Roof shell


Ship's hull

Definitions

Mechanics of Materials

Deals with

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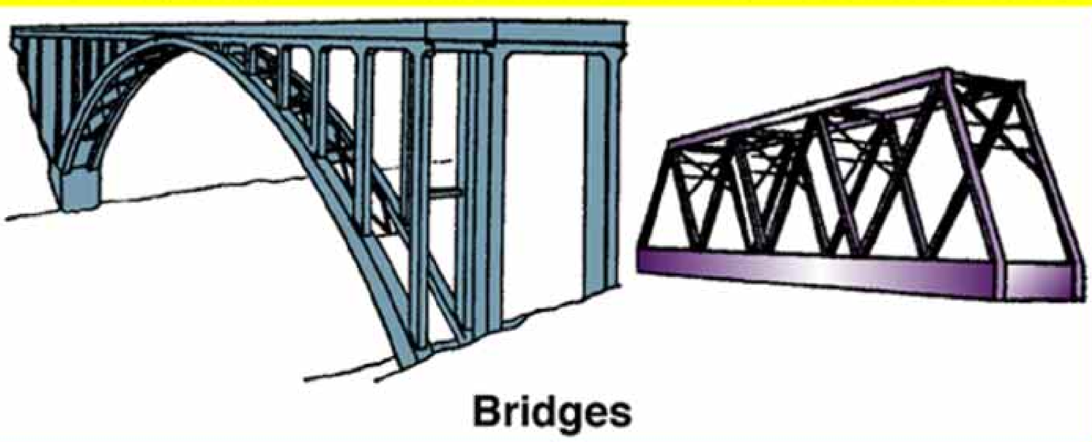
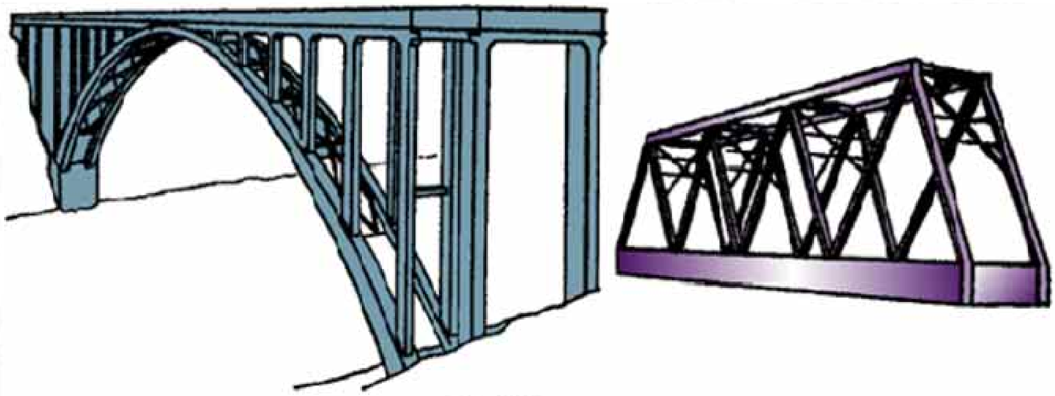

Bridges



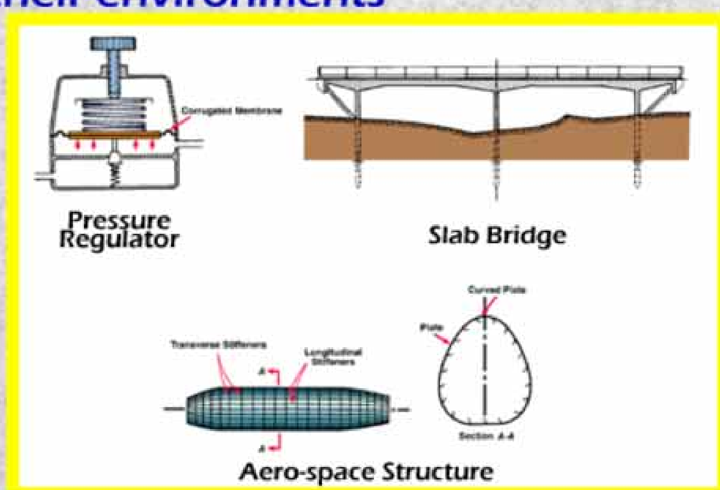
Bridges

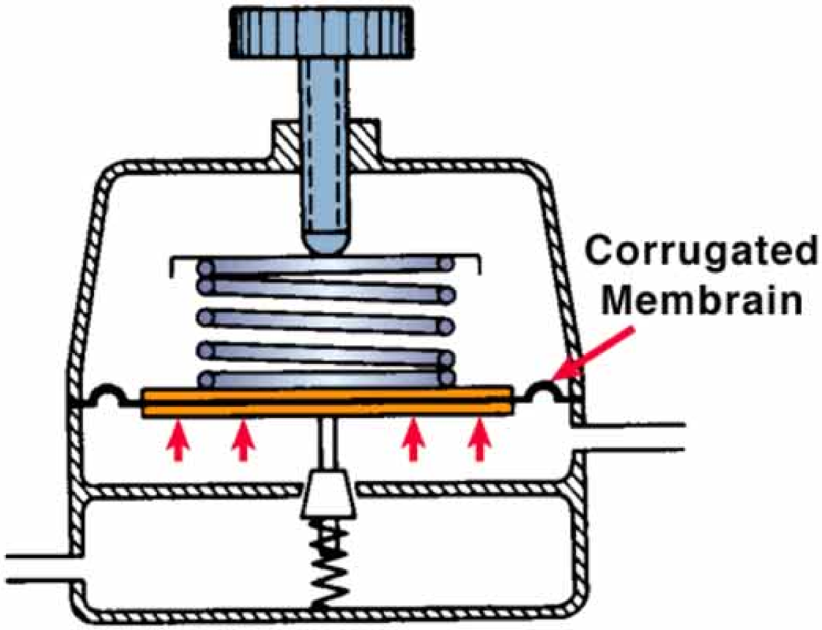
Definitions

Mechanics of Materials

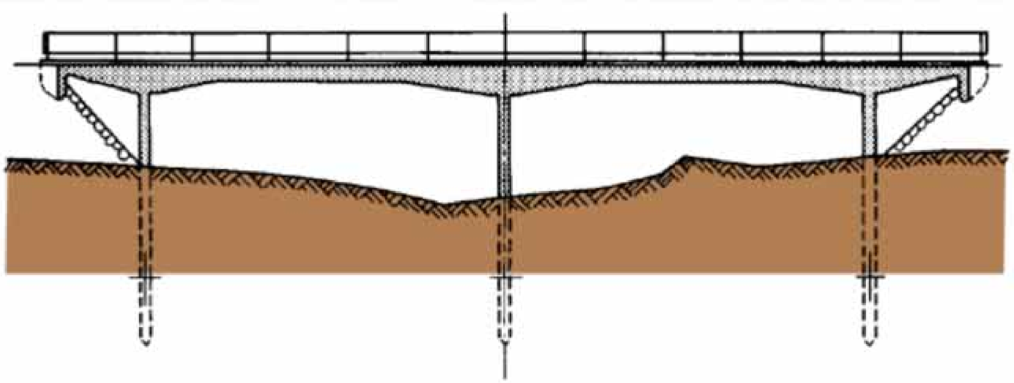
Deals with

Art of **idealizing** actual solids and structures and their environments

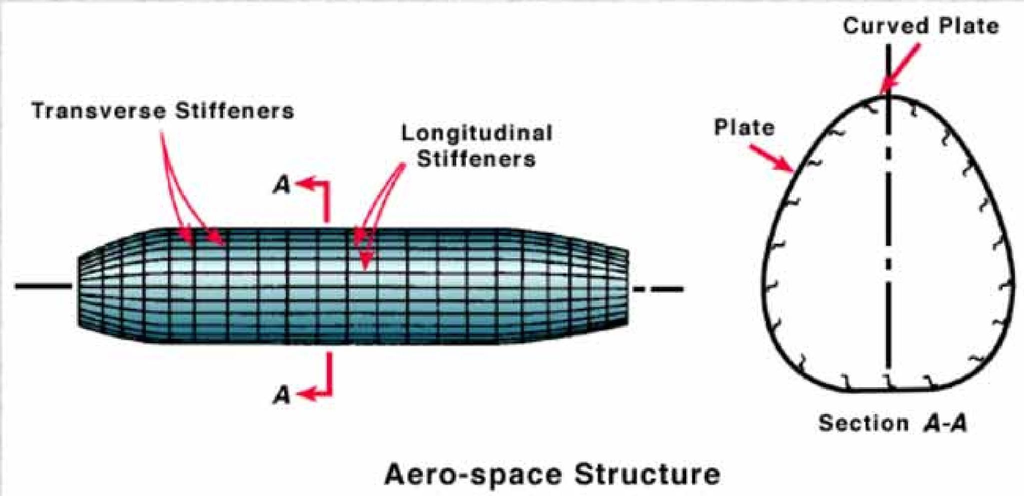




Pressure Regulator



Slab Bridge



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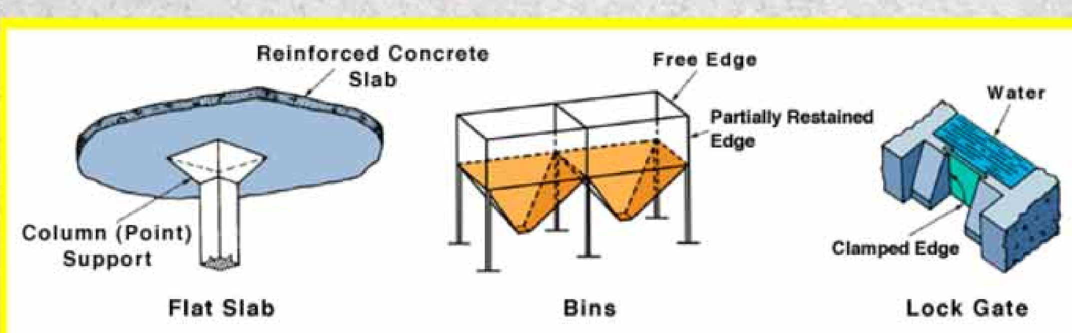

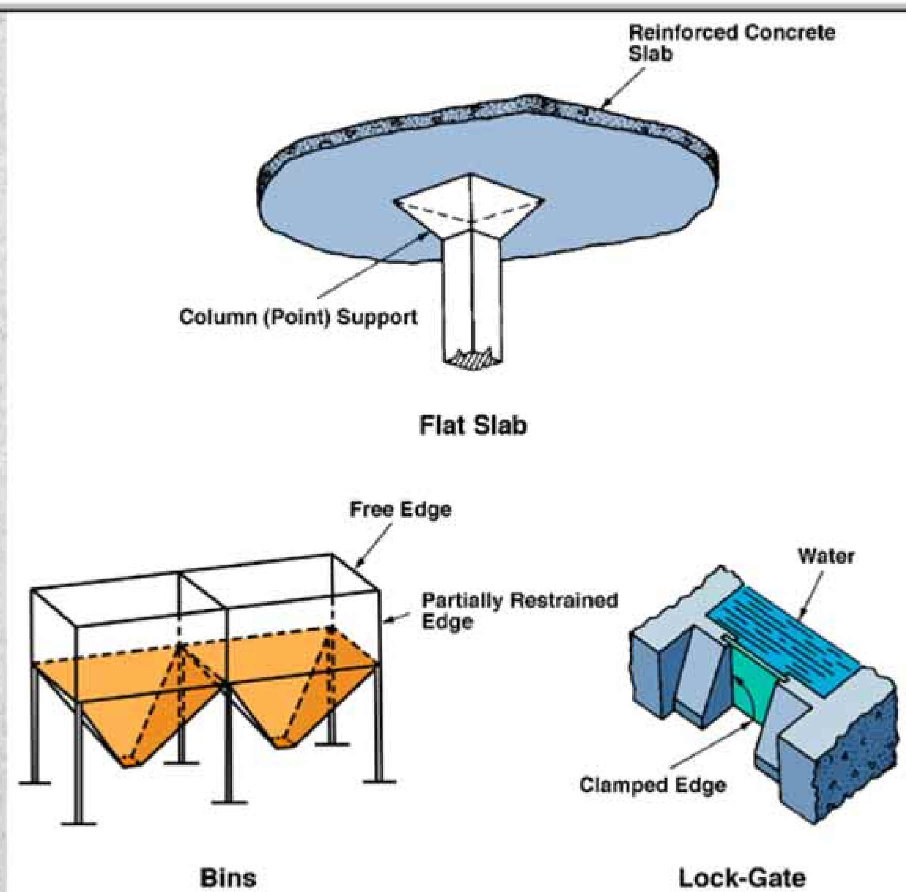

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Mechanics of Materials

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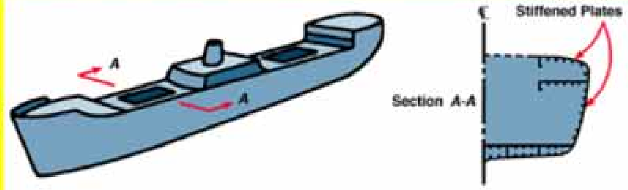
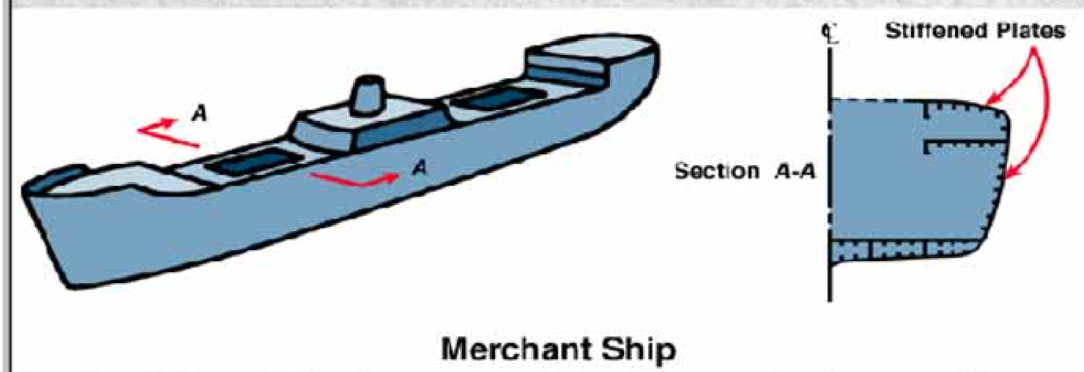


Merchant Ship



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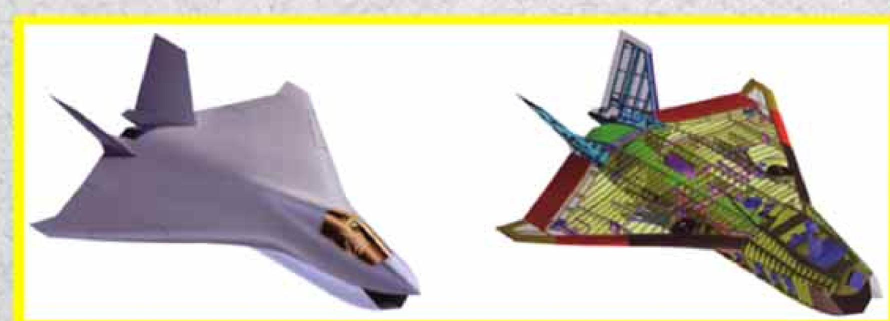

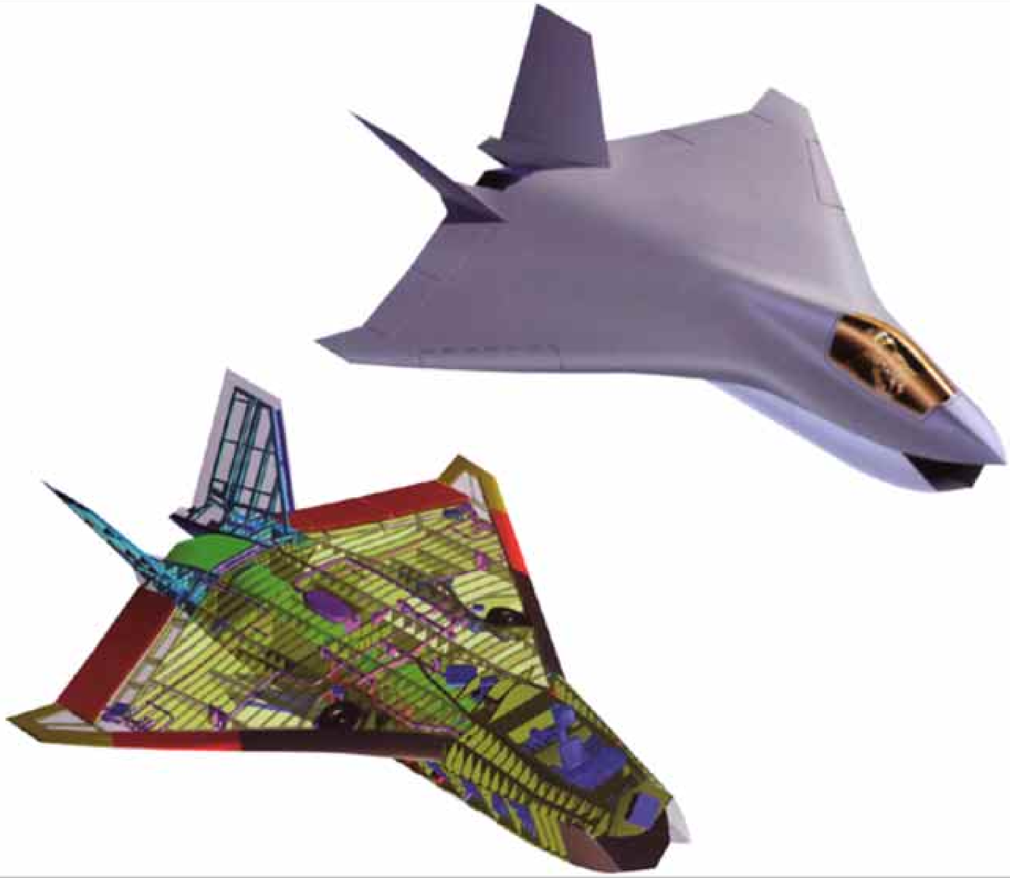

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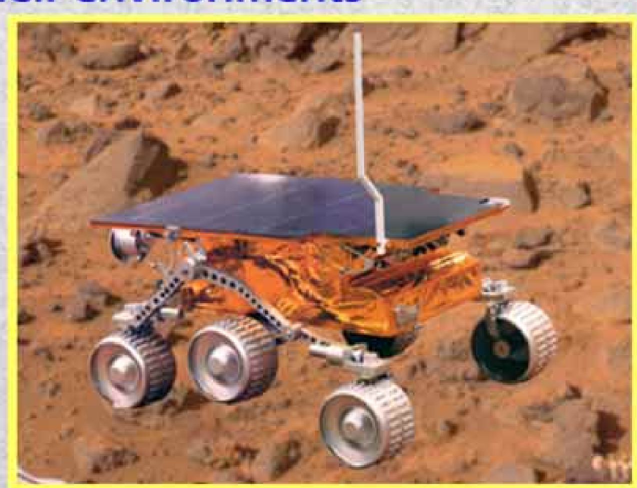

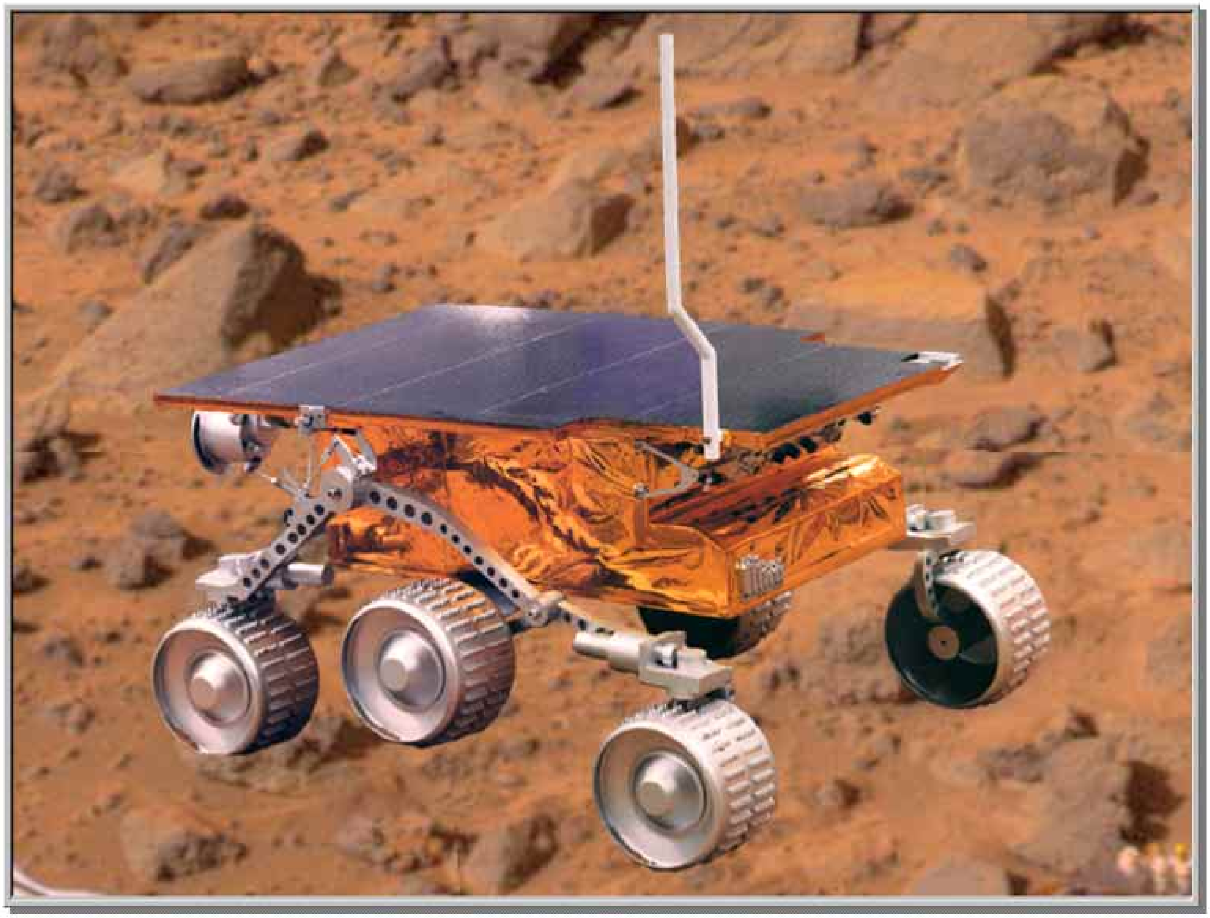

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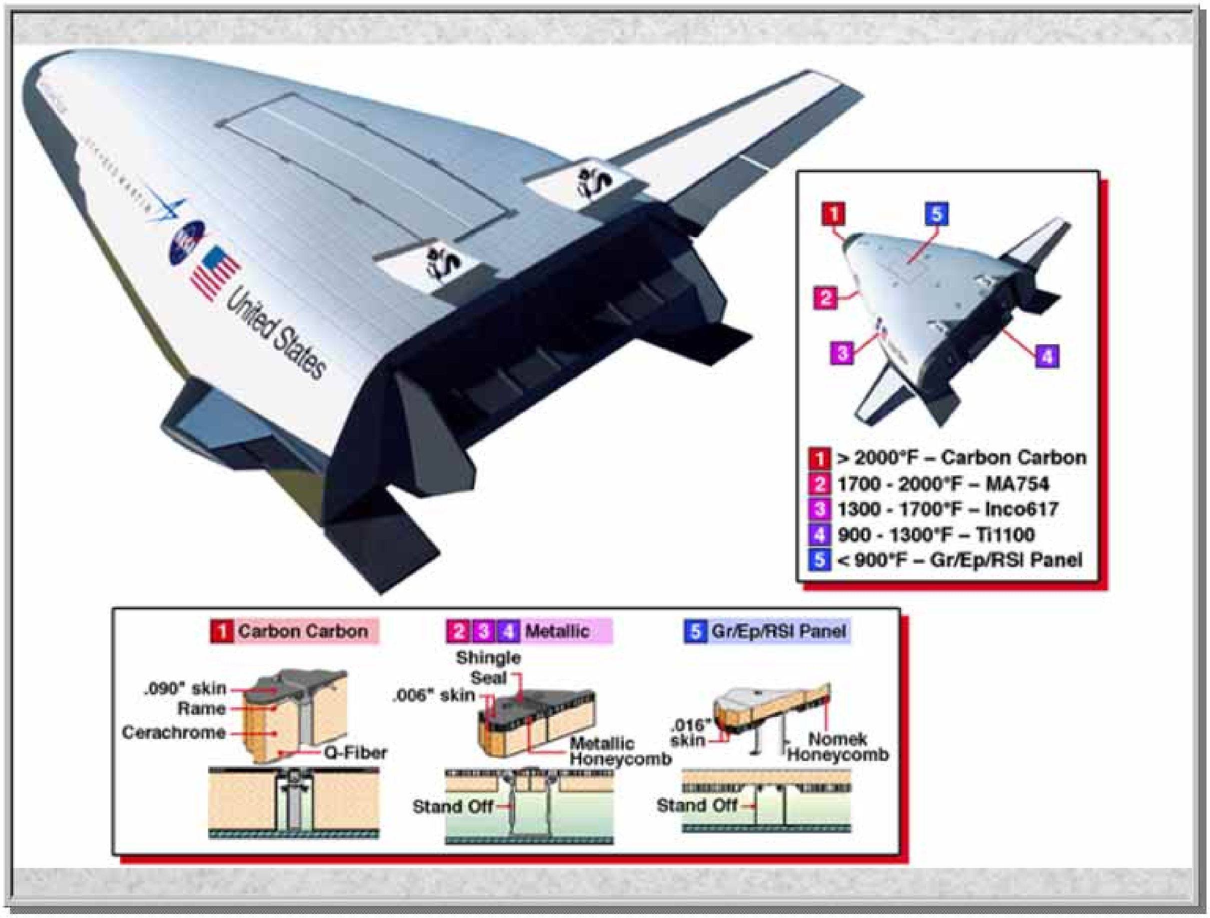

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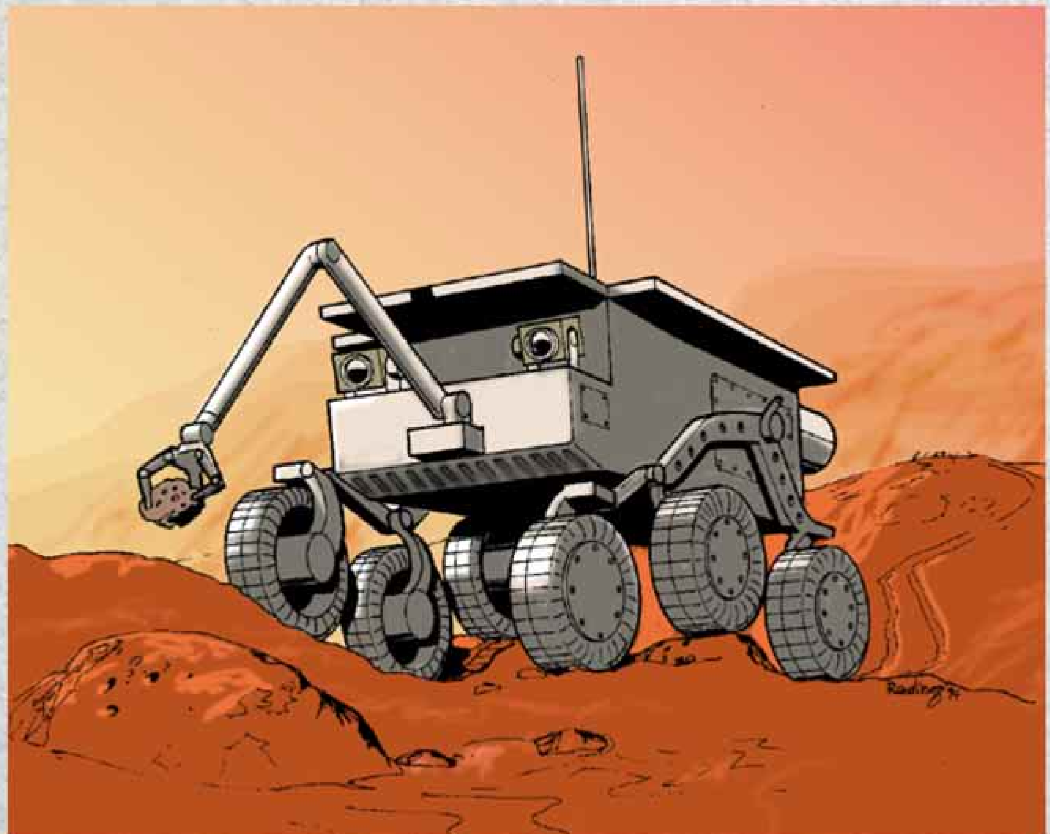


Definitions

Mechanics of Materials

Deals with

Prediction of *response, life* and *failure* of structures and components thereof using simplified theories.

Definitions

Response

Measured in terms of displacements, velocities, strains and stresses.

Definitions

Response

Functions which govern response can be grouped into:

Kinematic variables → **displacements, velocities, strains, strain rates**

Kinetic variables

Material characteristics

Source variables

Definitions

Response

Functions which govern response can be grouped into:

Kinematic variables

Kinetic variables → stresses, internal forces

Material characteristics

Source variables

Definitions

Response

Functions which govern response can be grouped into:

Kinematic variables

Kinetic variables

Material characteristics → stiffnesses, compliances, flexibilities

Source variables

Definitions

Response

Functions which govern response can be grouped into:

- Kinematic variables
- Kinetic variables
- Material characteristics
- Source variables → mechanical, environmental forces (mechanical, aerodynamic, thermal, optical and electromagnetic forces)

Relations Between External Forces and Response Quantities

Quantities

External Forces | Stresses | Strains | Displacements

Relations

Equations of equilibrium | Constitutive relations | Strain-displacement relations

Type of Relation

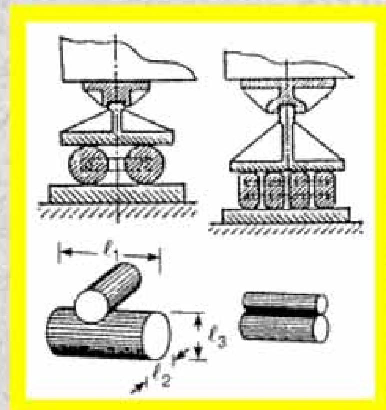
Physical Law | Semi-empirical -based on experiments | Geometric -based on logic

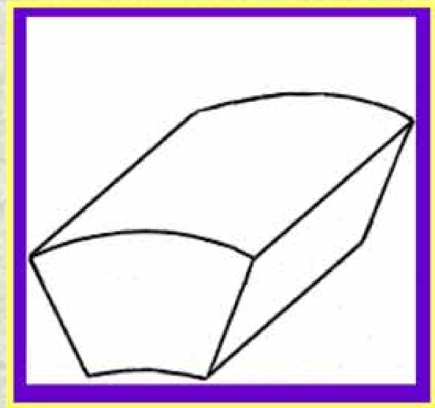
Continue

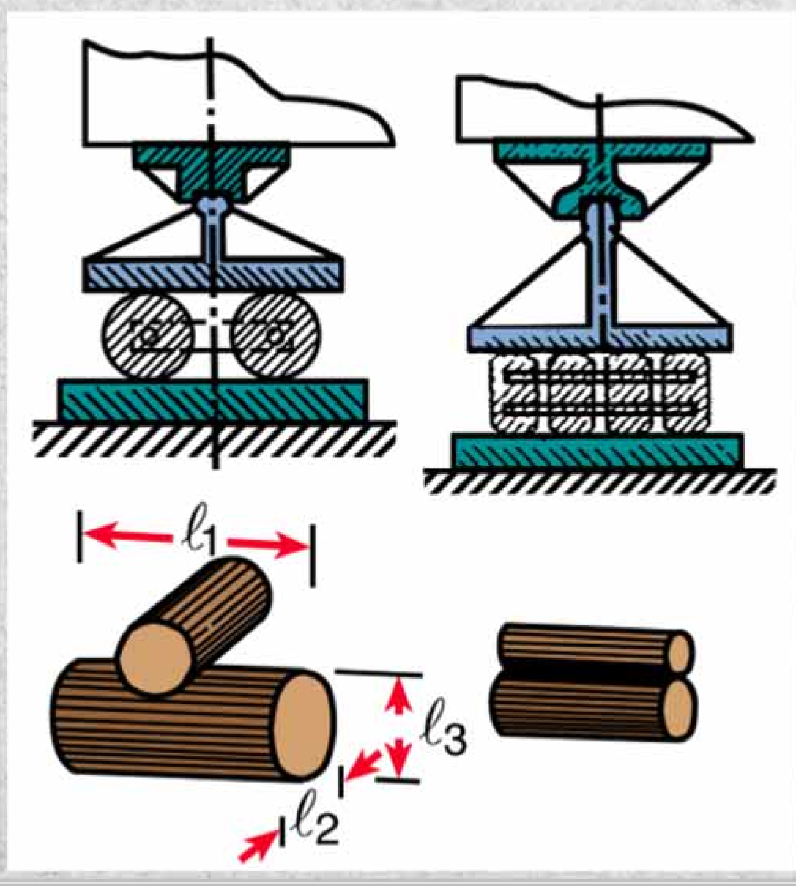
Classification of Structural Members (according to Spatial Extent)

Three-dimensional members

$$\ell_1 = O(\ell_2) = O(\ell_3)$$







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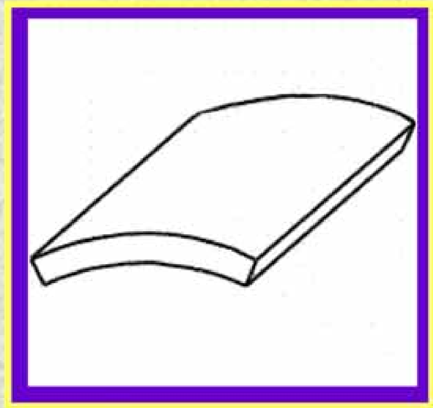
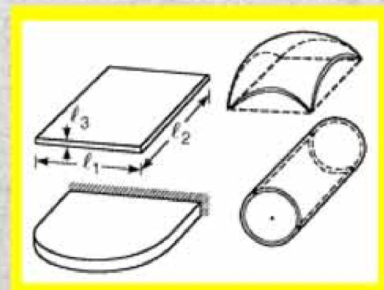
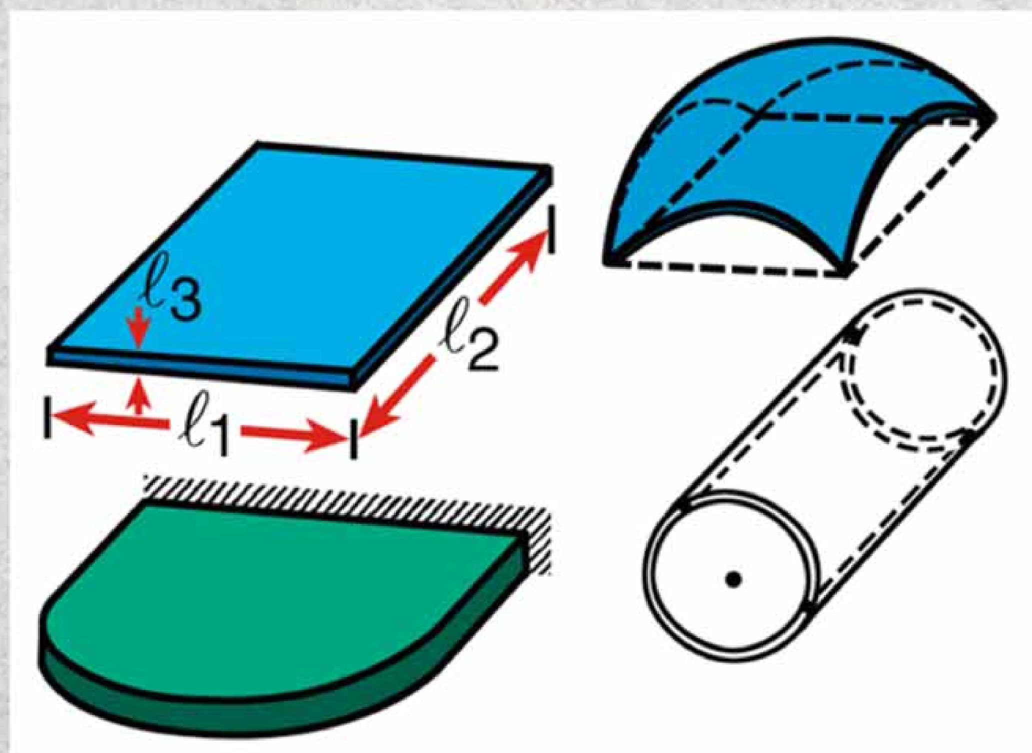


Two-dimensional members

$$\ell_1 = O(\ell_2) >> \ell_3$$





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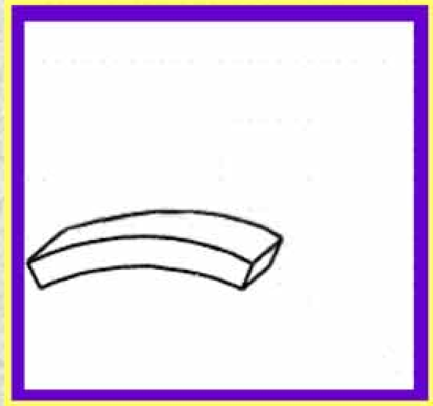
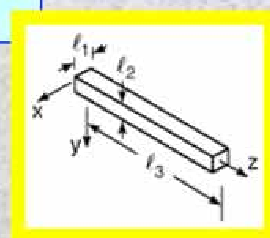
Two-dimensional members

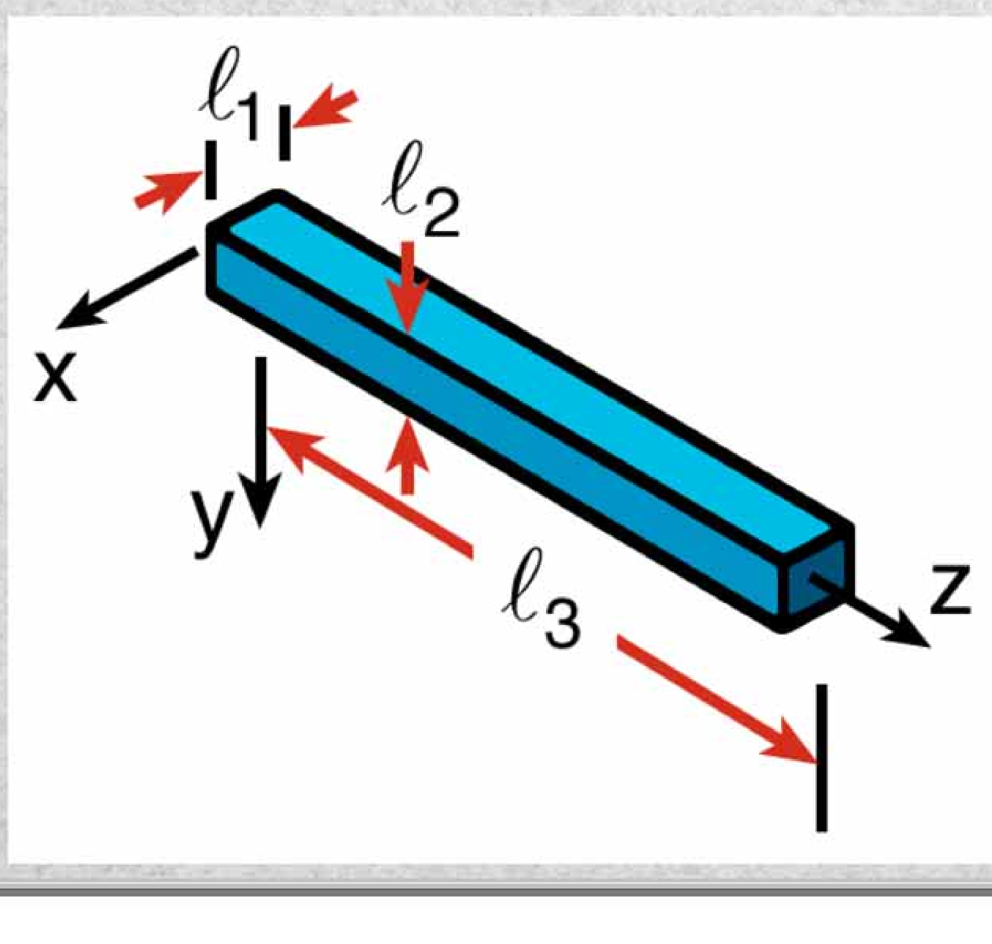
$$\ell_1 = O(\ell_2) >> \ell_3$$



One-dimensional members

$$\ell_2 = O(\ell_1) << \ell_3$$





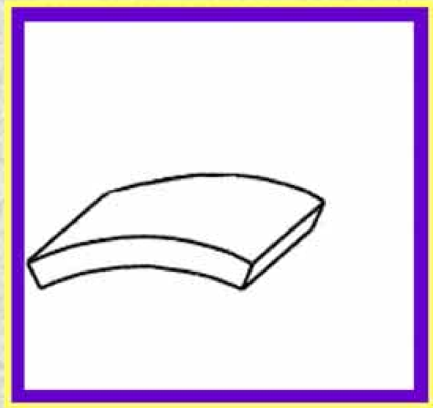
Classification of Structural Members (according to Spatial Extent)

Three-dimensional members

$\ell_1 = O(\ell_2) = O(\ell_3)$

Two-dimensional members

$\ell_1 = O(\ell_2) >> \ell_3$

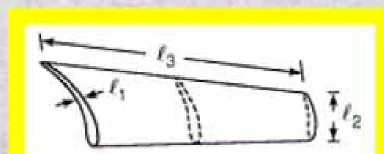


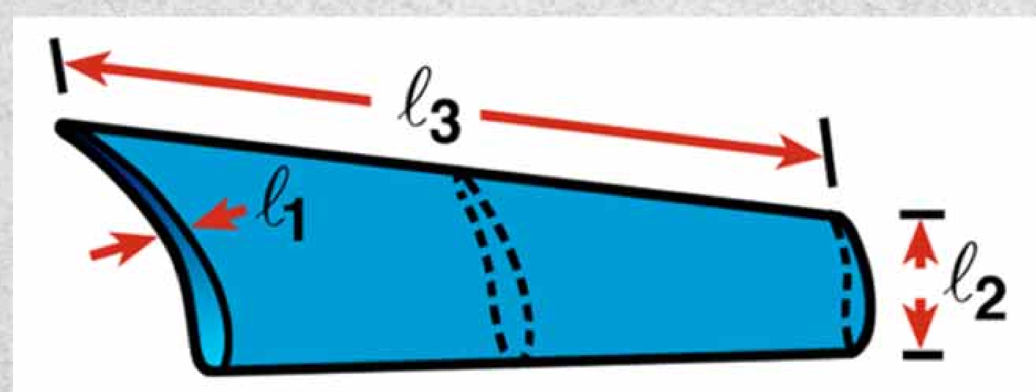
One-dimensional members

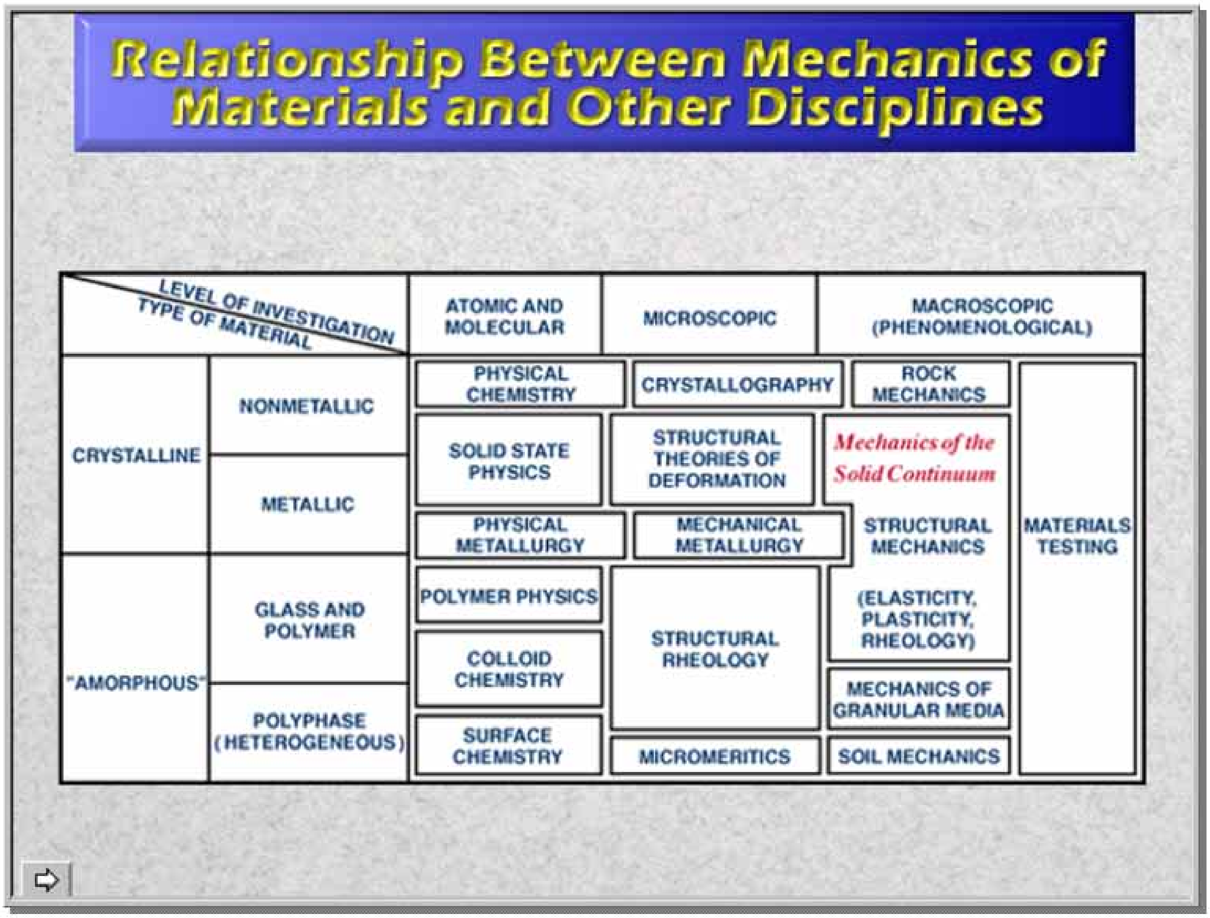
$\ell_2 = O(\ell_1) << \ell_3$

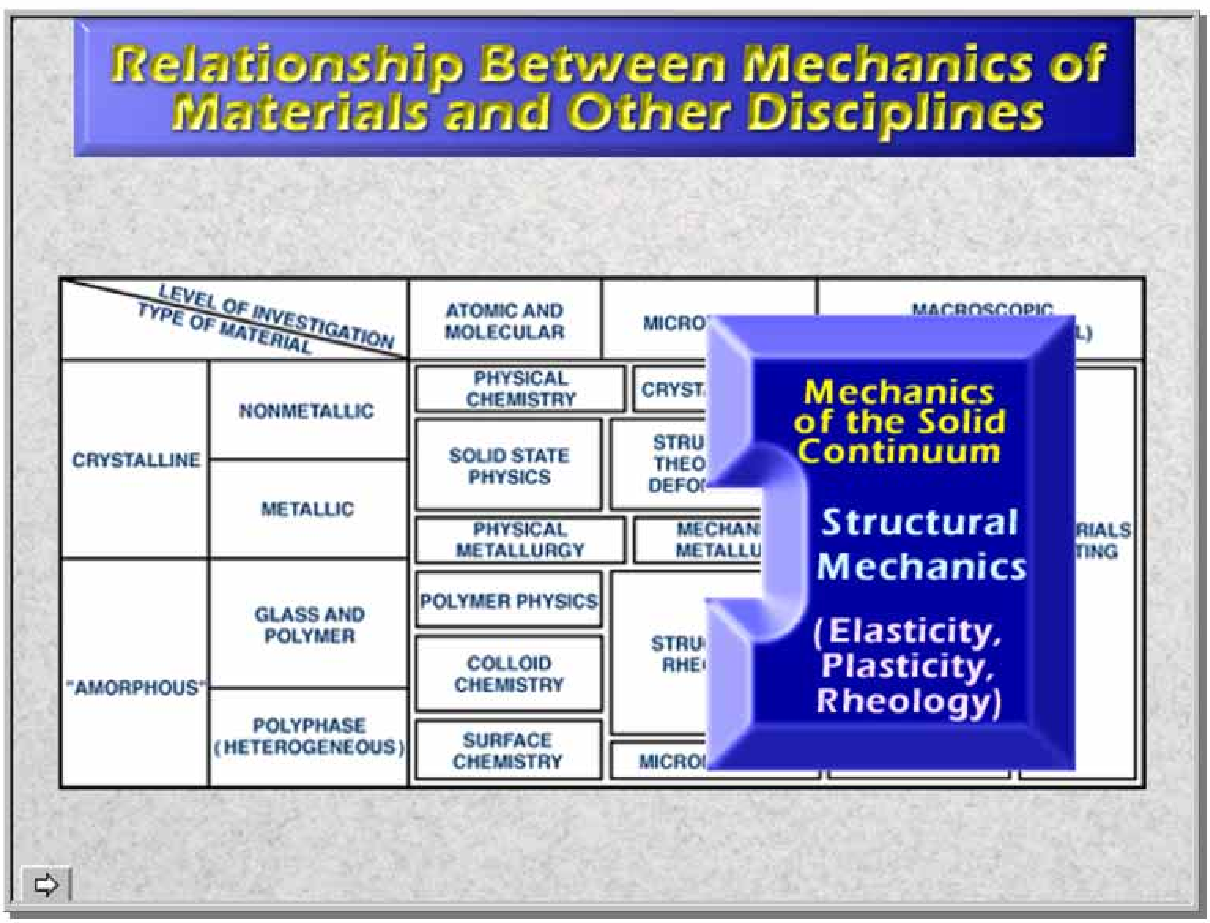
Thin-walled beams

$\ell_3 >> \ell_2 >> \ell_1$






Relationship Between Mechanics of Materials and Other Disciplines
LEVEL OF INVESTIGATION
TYPE OF MATERIAL
ATOMIC AND MOLECULAR
MICROSCOPIC
MACROSCOPIC (PHENOMENOLOGICAL)
CRYSTALLINE
NONMETALLIC
METALLIC
"AMORPHOUS"
GLASS AND POLYMER
POLYPHASE (HETEROGENEOUS)
PHYSICAL CHEMISTRY
CRYSTALLOGRAPHY
ROCK MECHANICS
SOLID STATE PHYSICS
STRUCTURAL THEORIES OF DEFORMATION
Mechanics of the Solid Continuum
PHYSICAL METALLURGY
MECHANICAL METALLURGY
STRUCTURAL MECHANICS
MATERIALS TESTING
POLYMER PHYSICS
(ELASTICITY, PLASTICITY, RHEOLOGY)
STRUCTURAL RHEOLOGY
COLLOID CHEMISTRY
MECHANICS OF GRANULAR MEDIA
SURFACE CHEMISTRY
MICROMERITICS
SOIL MECHANICS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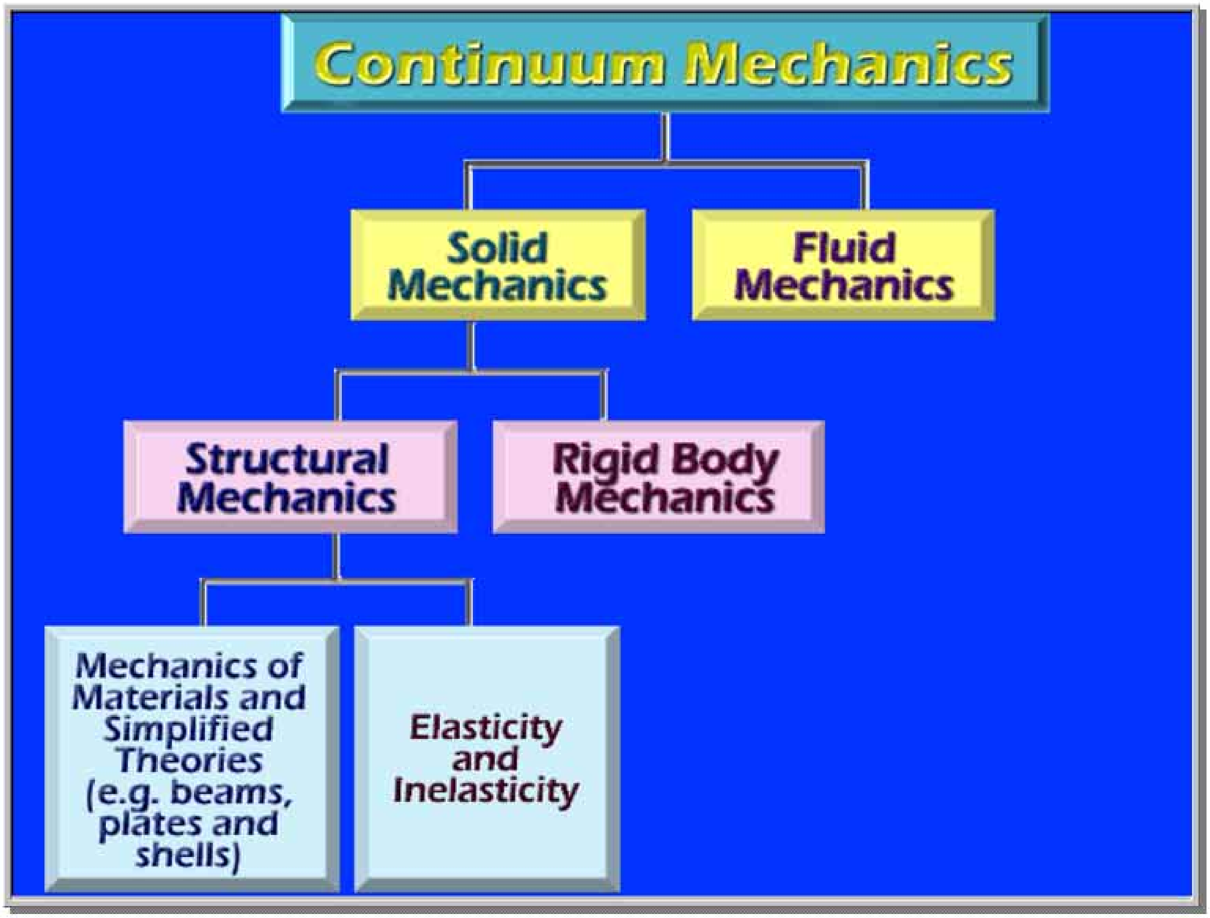
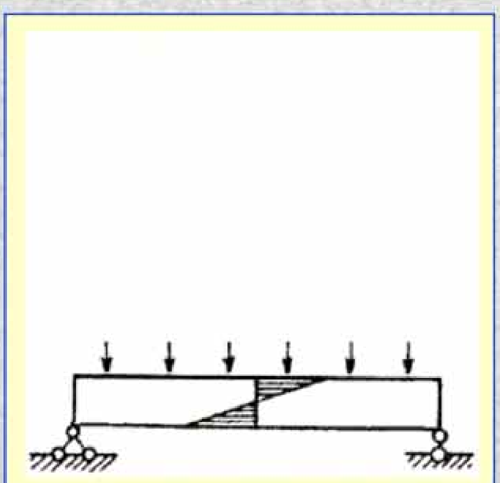
PHYSICAL CHEMISTRY
SOLID STATE PHYSICS
PHYSICAL METALLURGY
POLYMER PHYSICS
COLLOID CHEMISTRY
SURFACE CHEMISTRY
Mechanics of the Solid Continuum
Structural Mechanics
(Elasticity, Plasticity, Rheology)



Mechanics of Materials vs. Elasticity and Inelasticity

Examples of problems for which mechanics of materials assumptions are not valid



Shallow Beams

Mechanics of Materials vs. Elasticity and Inelasticity

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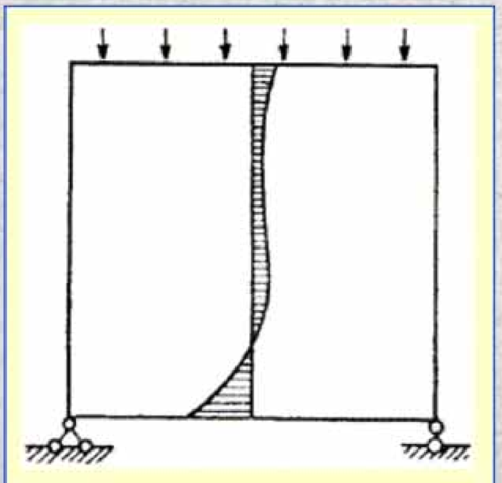

Deep Beams

Mechanics of Materials vs. Elasticity and Inelasticity

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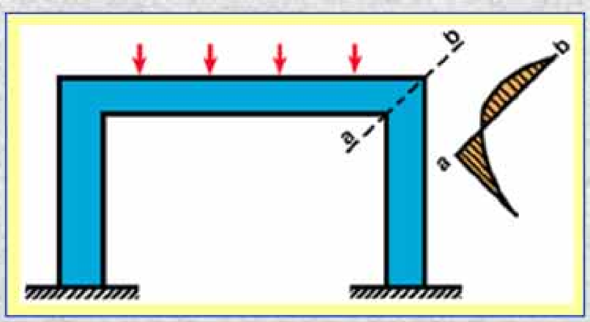
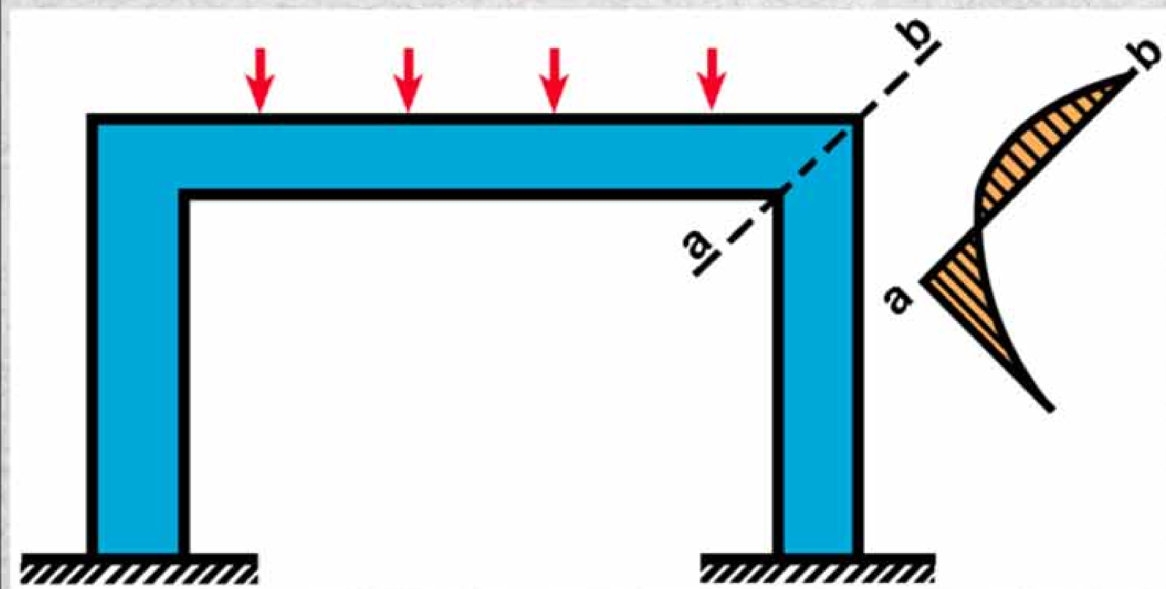


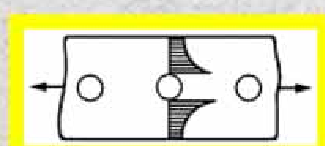
Stresses at frame joints



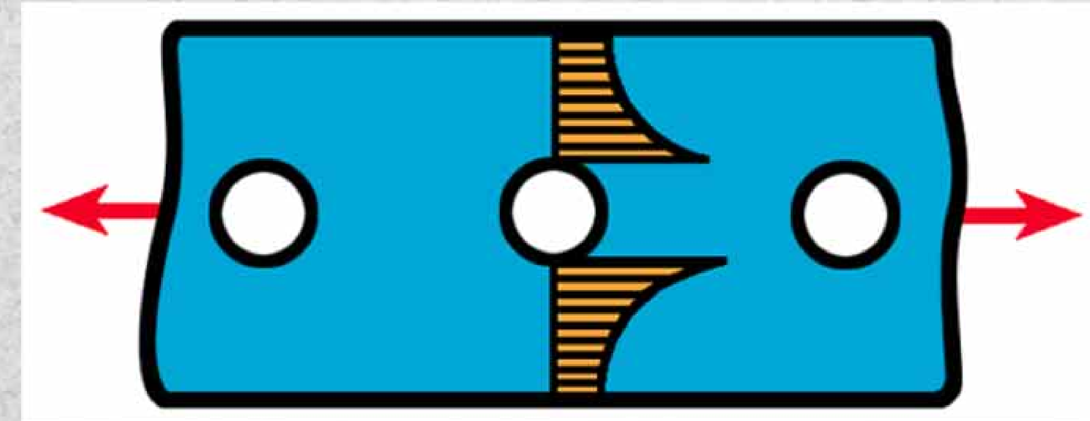
Mechanics of Materials vs. Elasticity and Inelasticity

Examples of problems for which mechanics of materials assumptions are not valid

Stress concentrations



- Near discontinuities (cutouts and sharp changes)
- Near points of application of loads



Mechanics of Materials vs. Elasticity and Inelasticity

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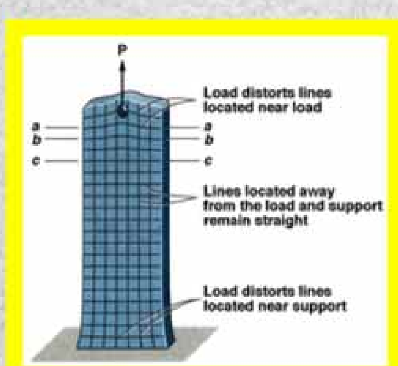

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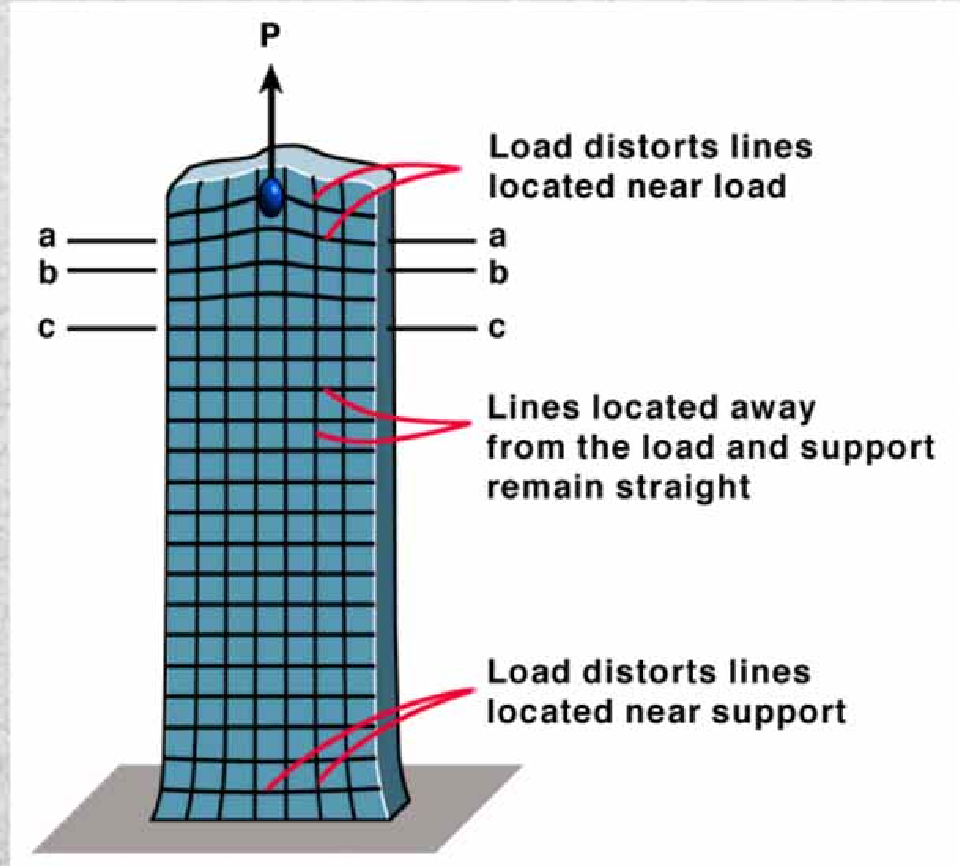

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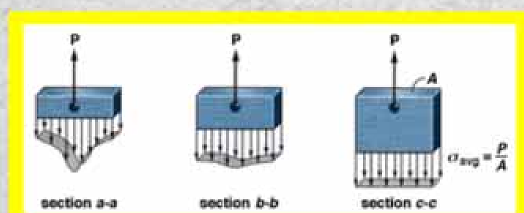

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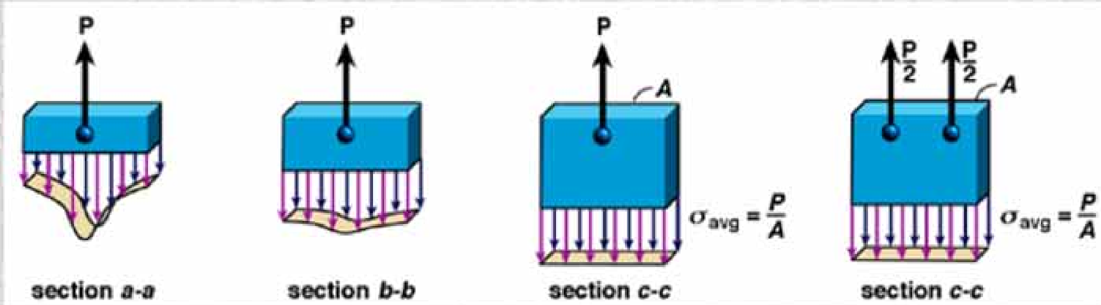
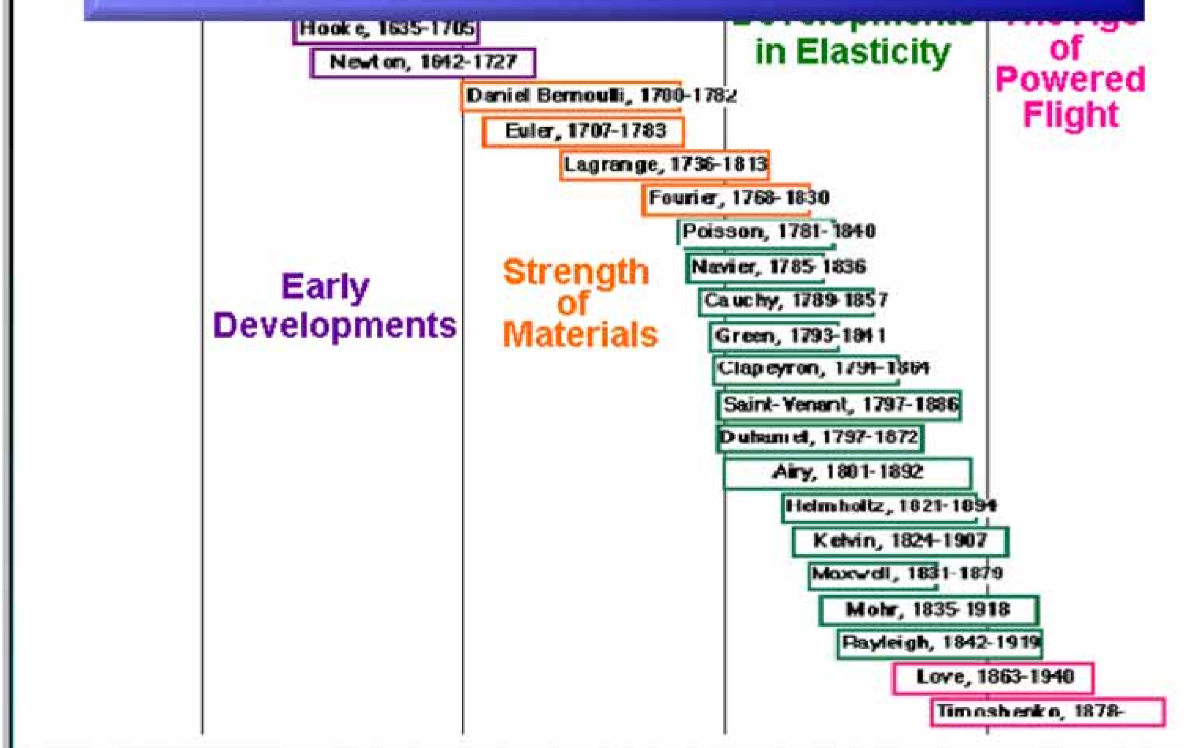
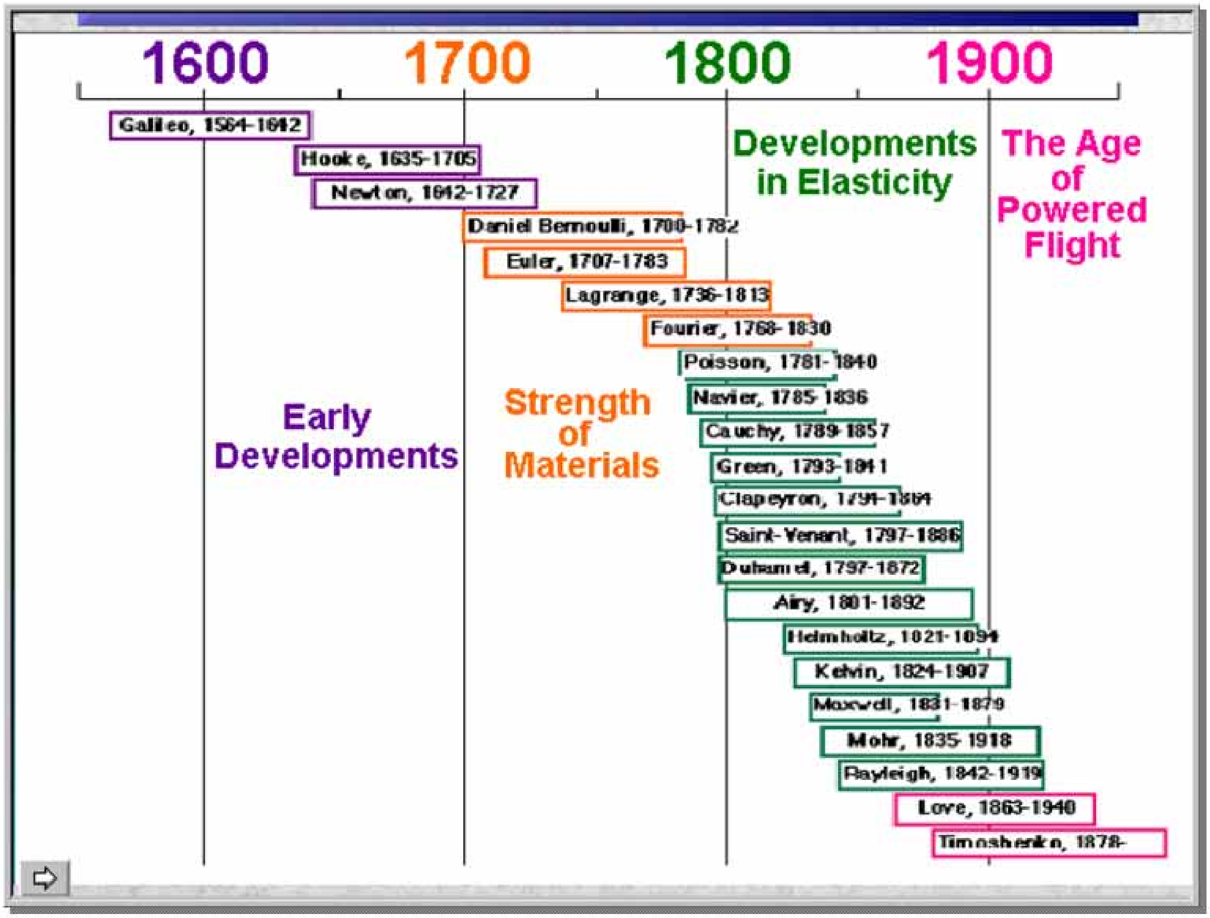


Brief History of the Development of Mechanics of Materials

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1700
1800
1900
Galileo, 1564-1642
Hooke, 1635-1705
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Daniel Bernoulli, 1700-1782
Euler, 1707-1783
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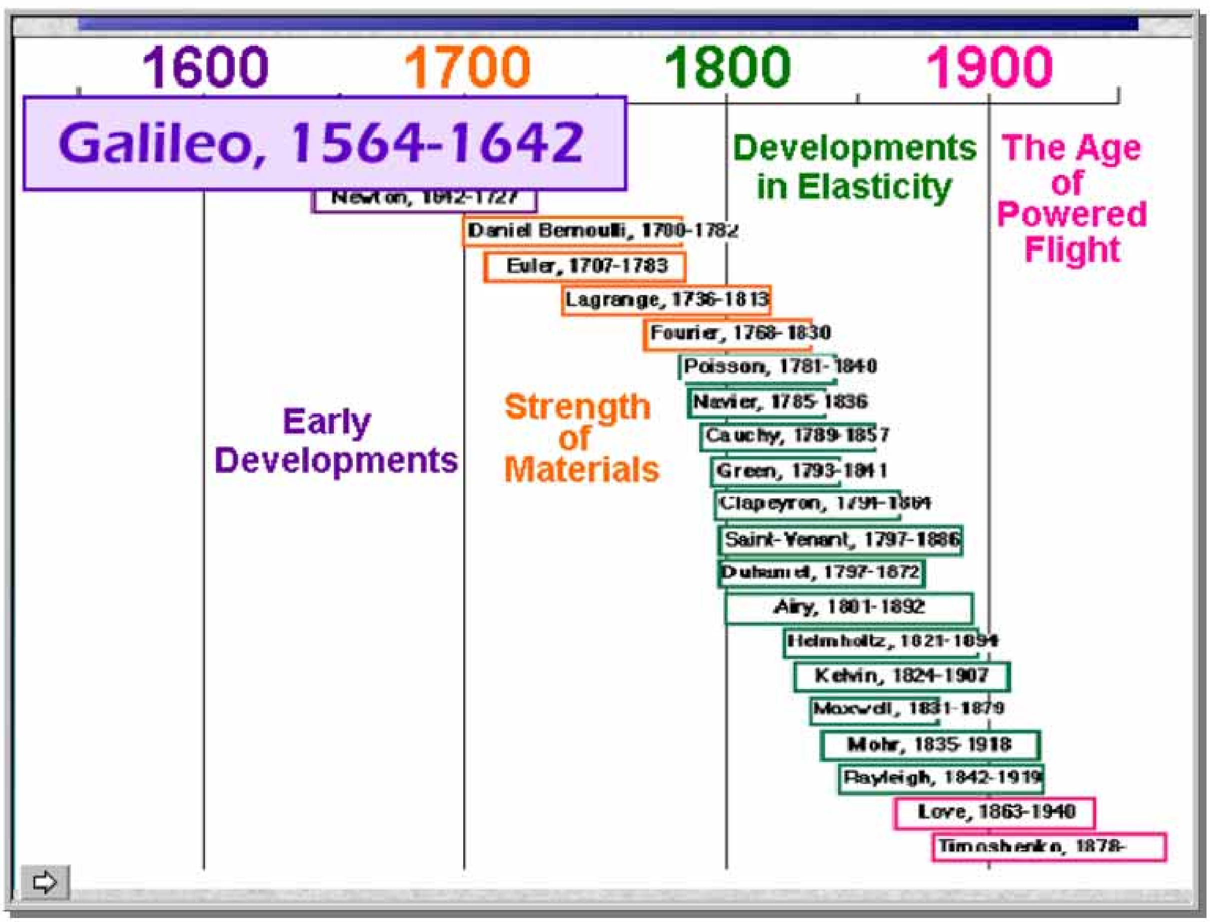
Early Developments
Strength of Materials
Developments in Elasticity
The Age of Powered Flight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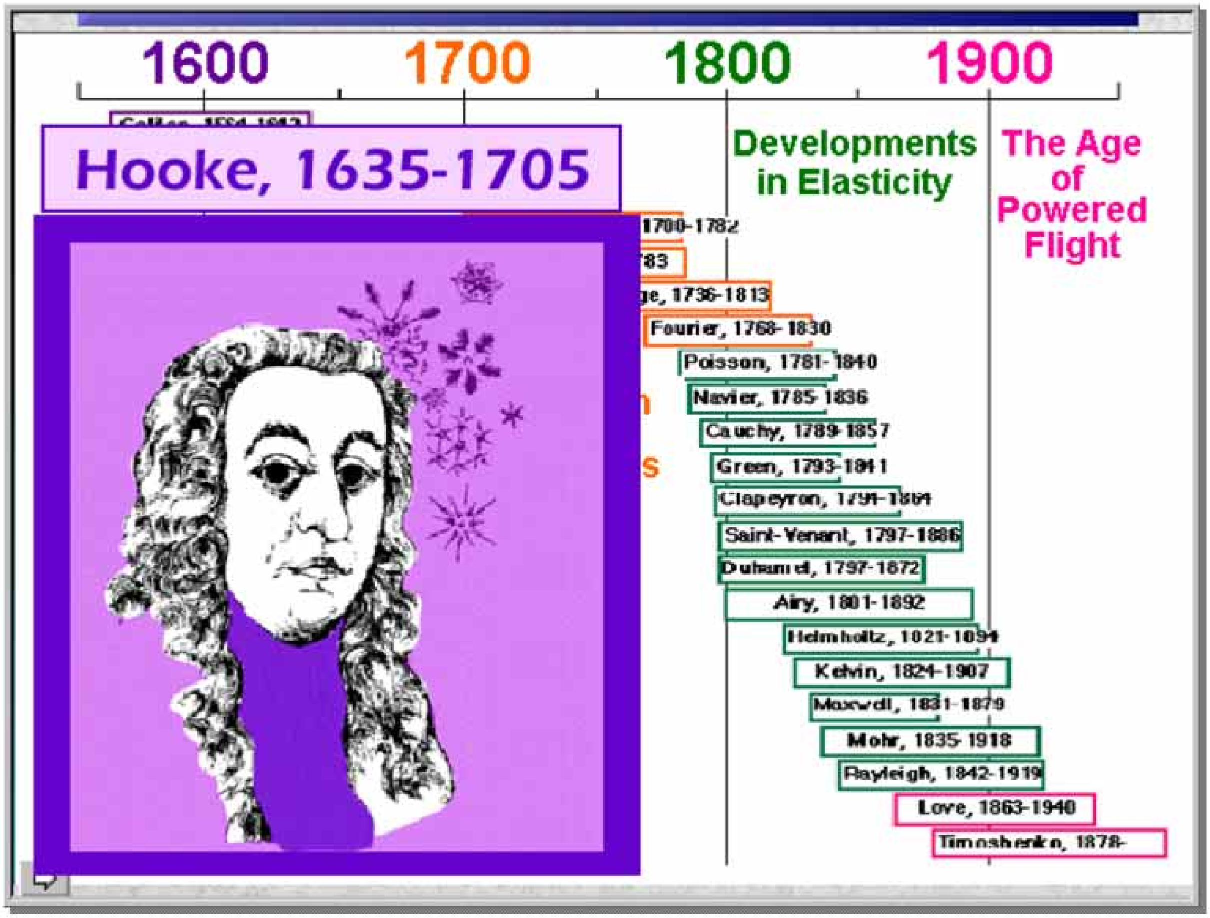
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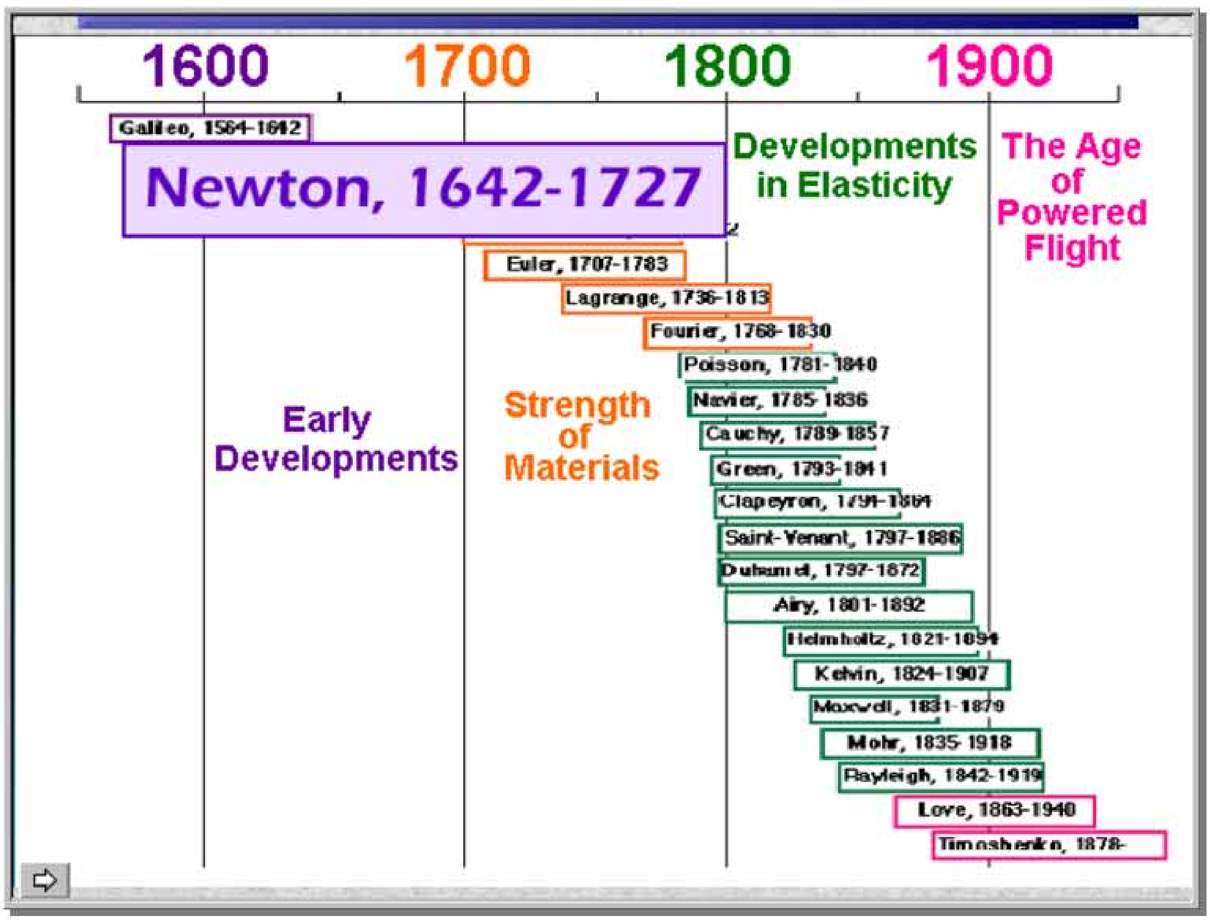
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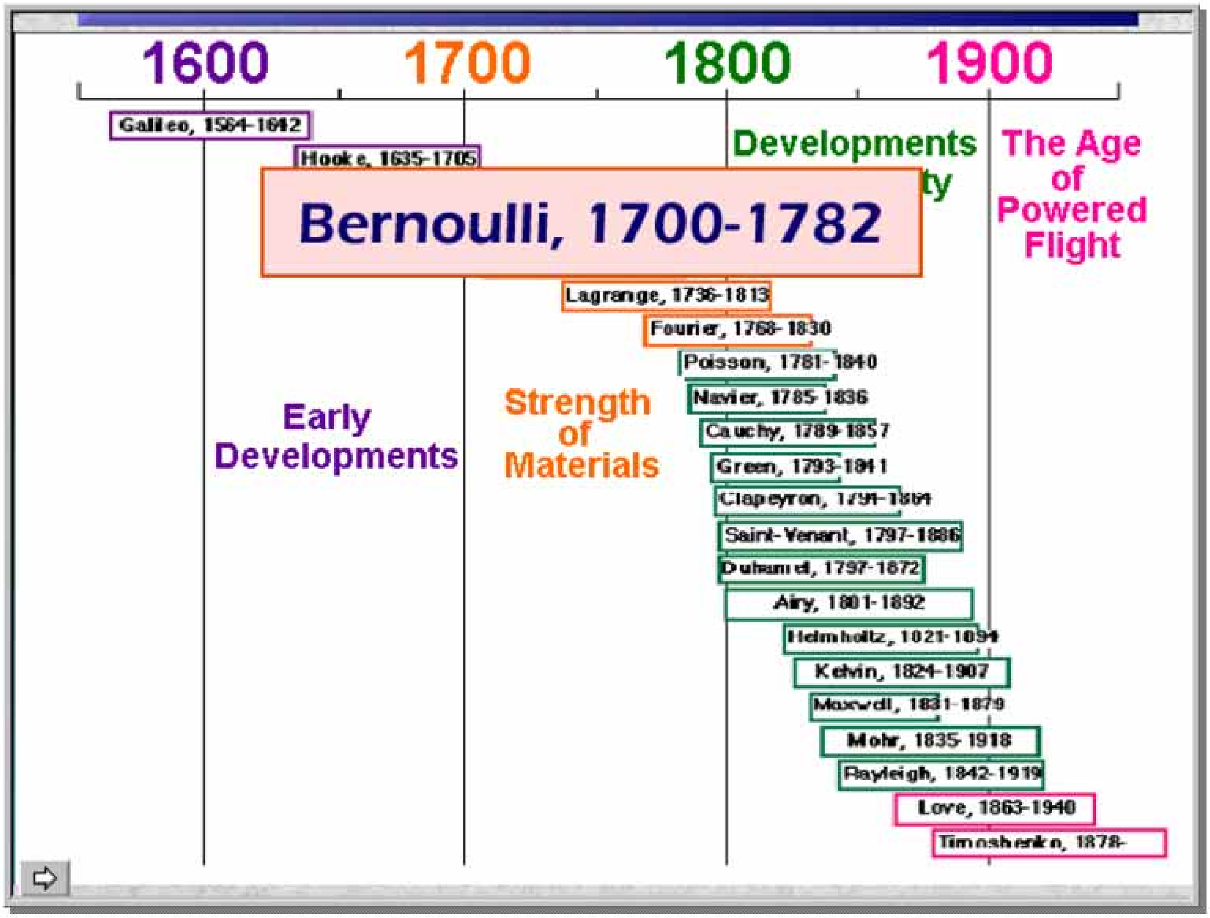
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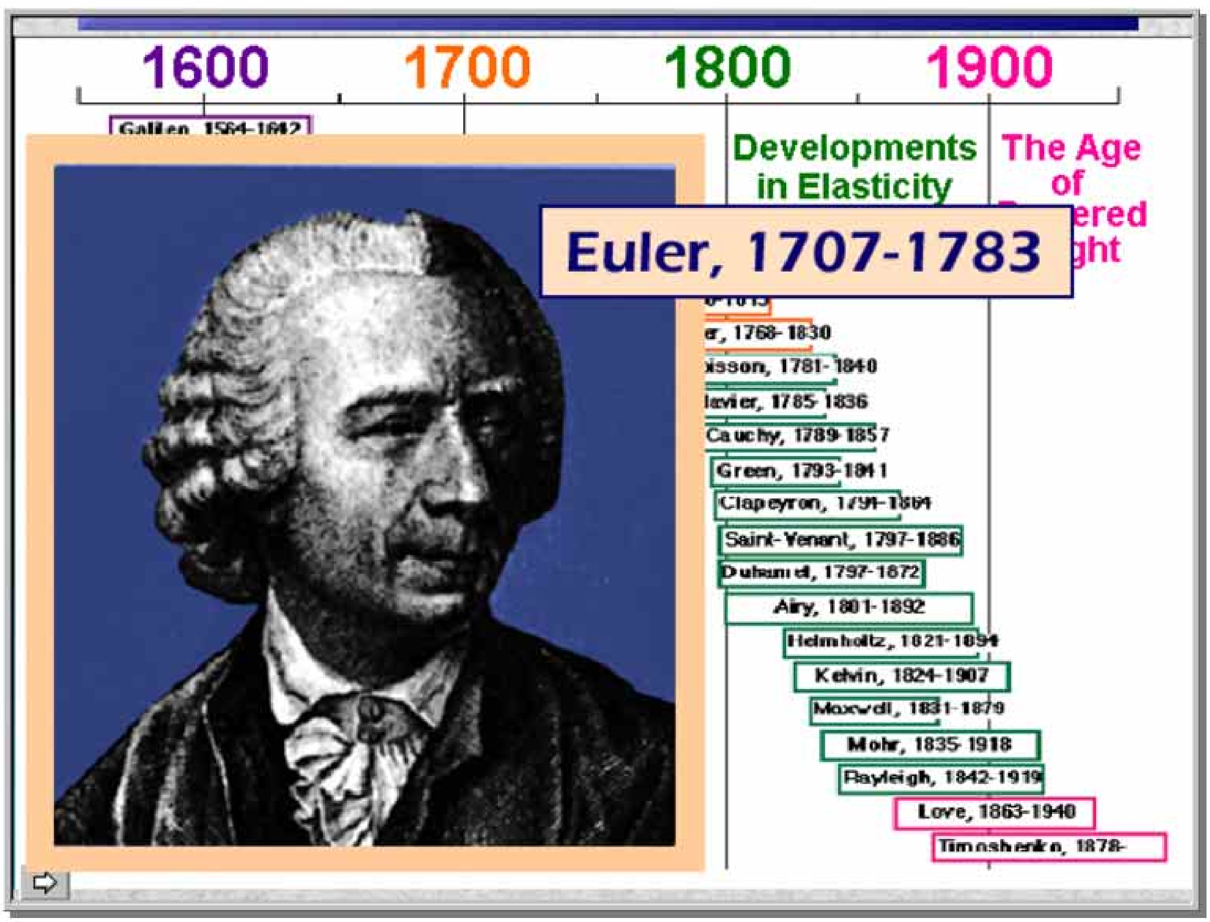
Strength of Materials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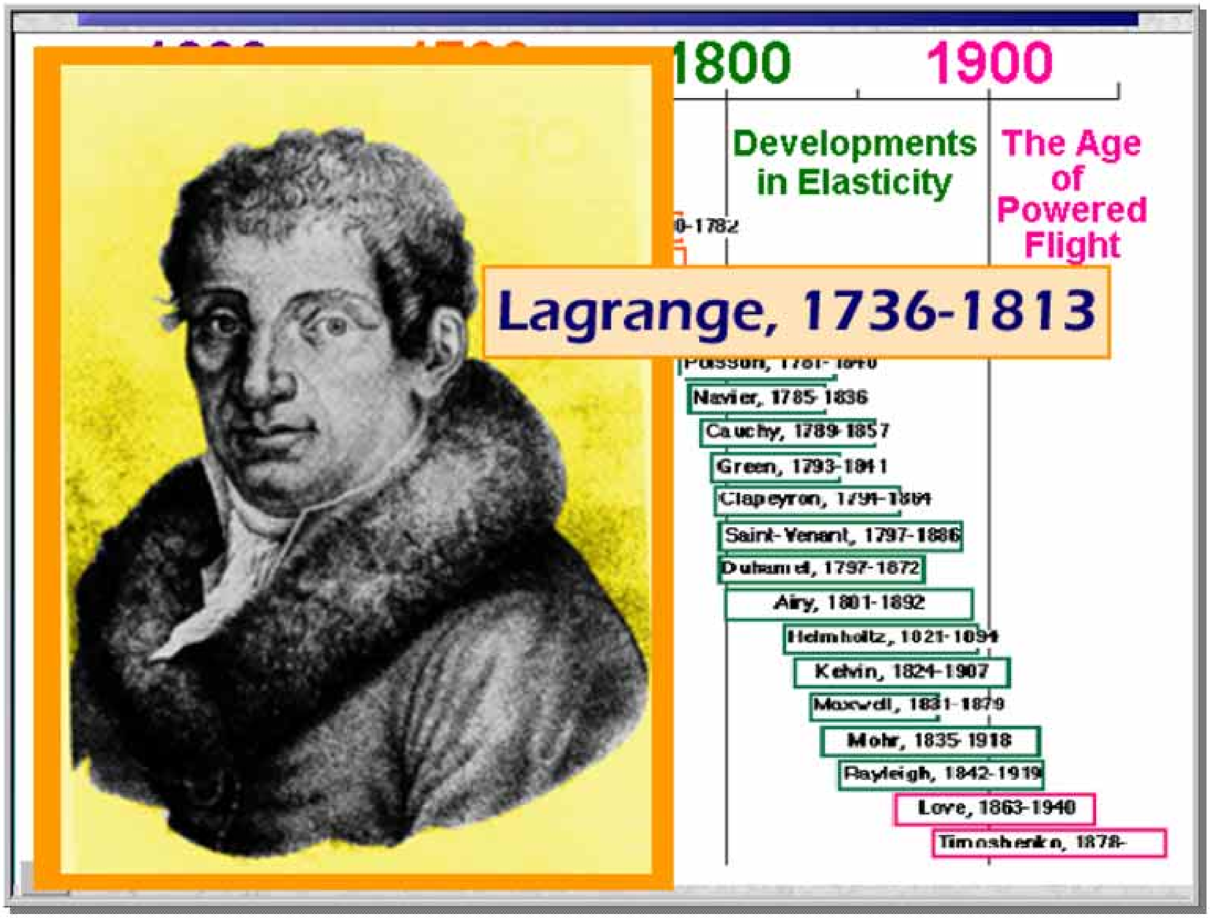
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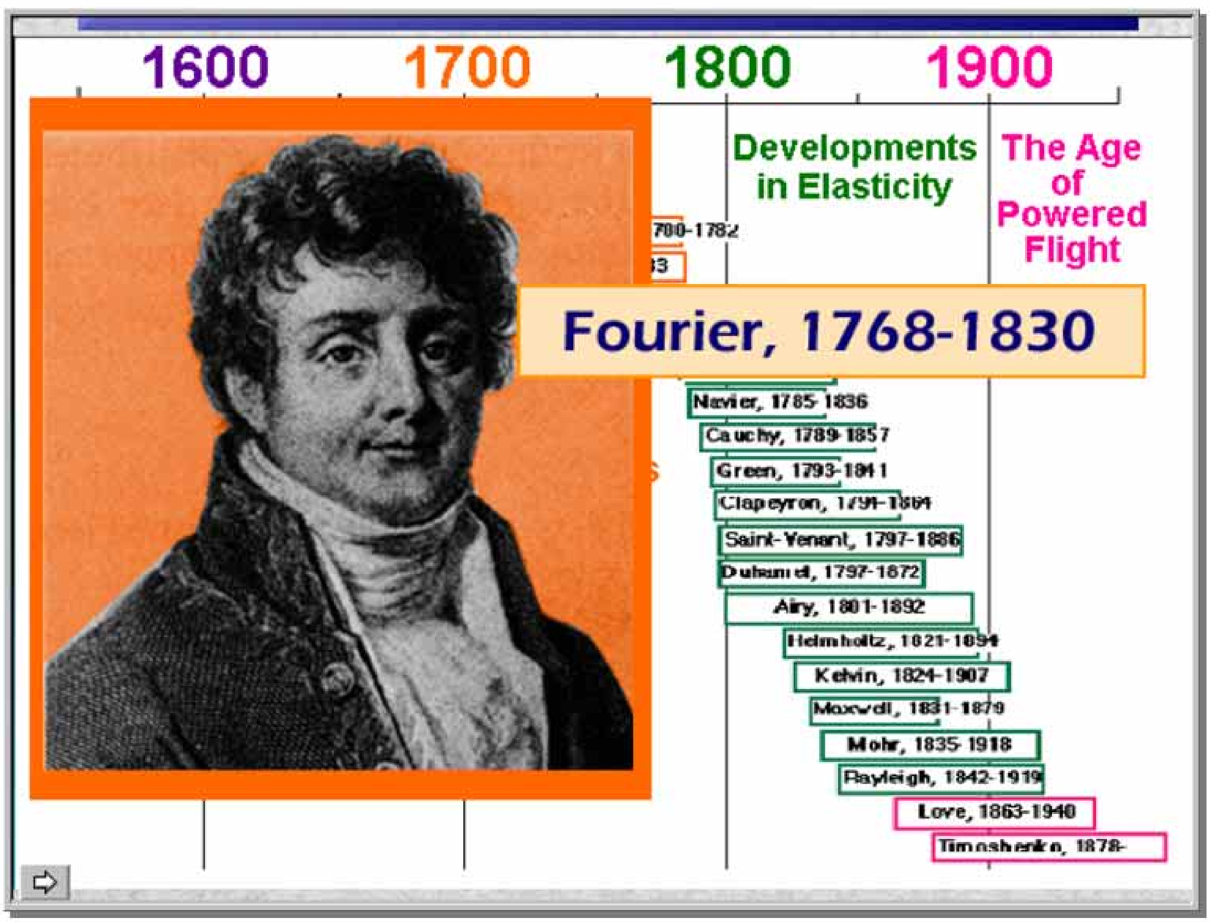
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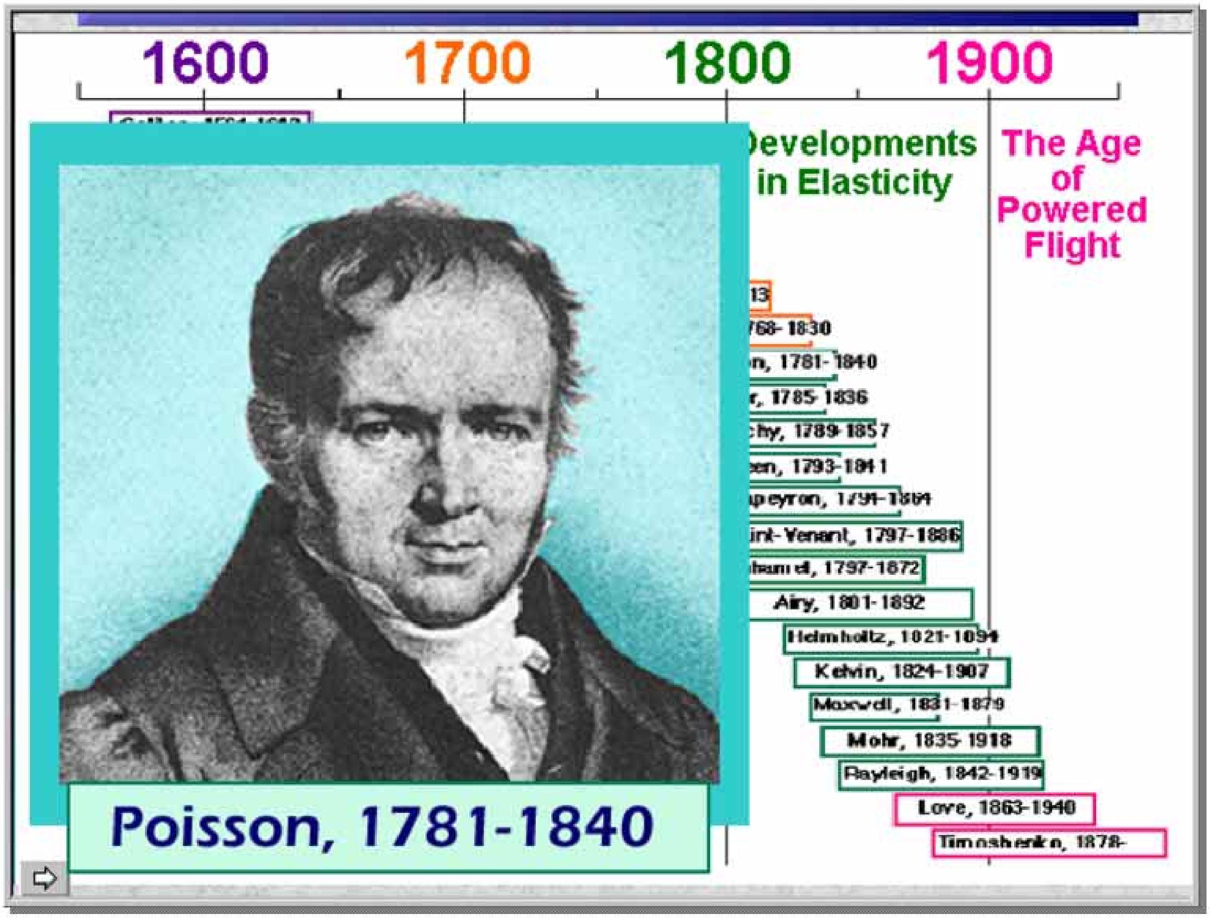
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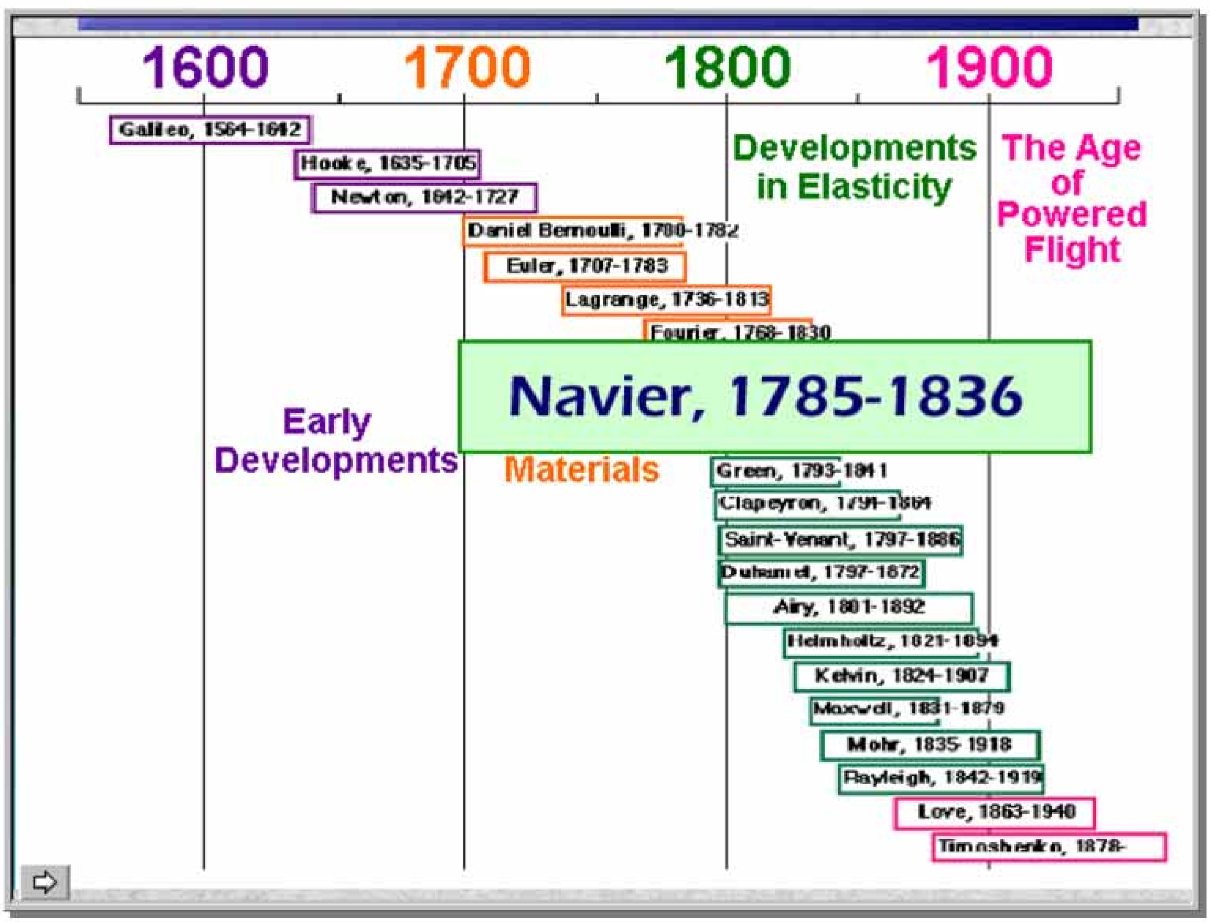
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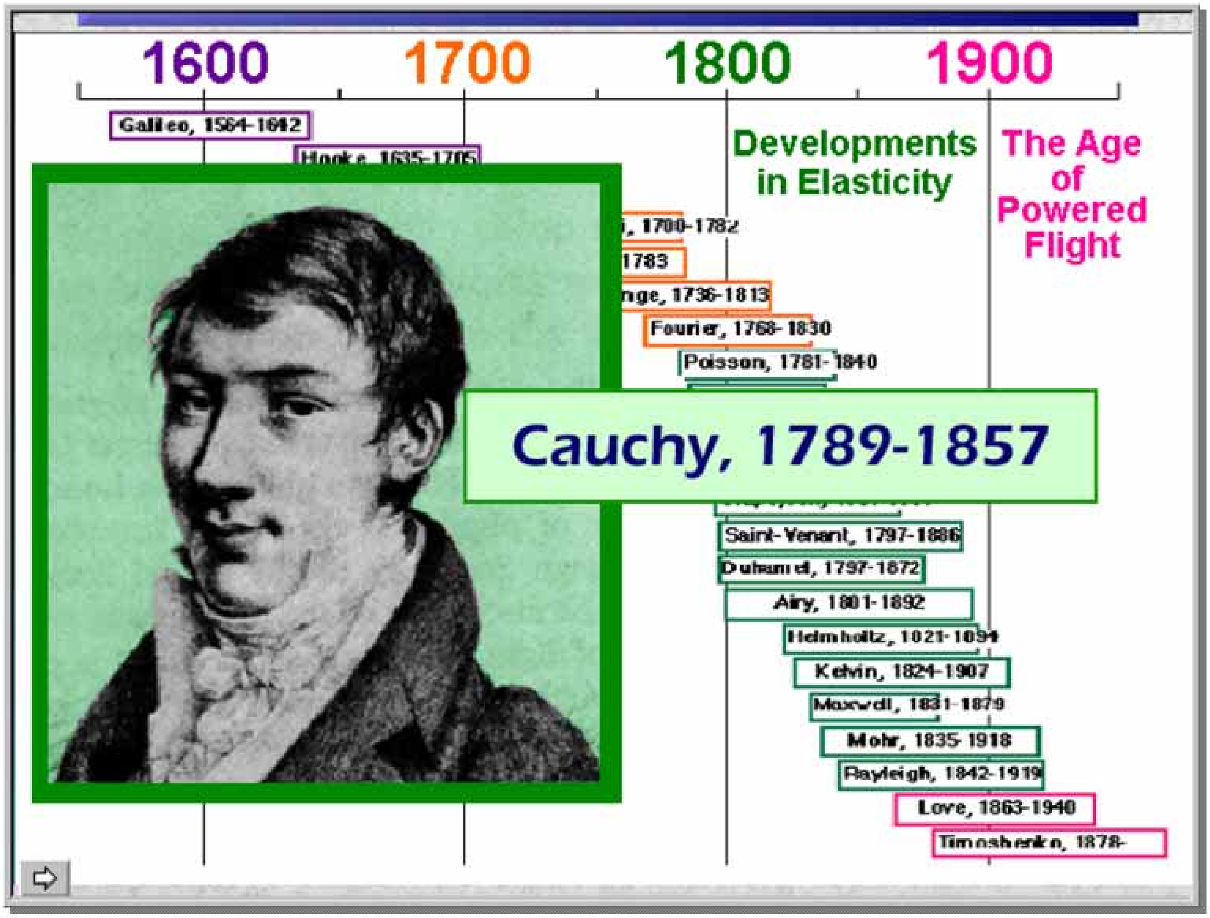
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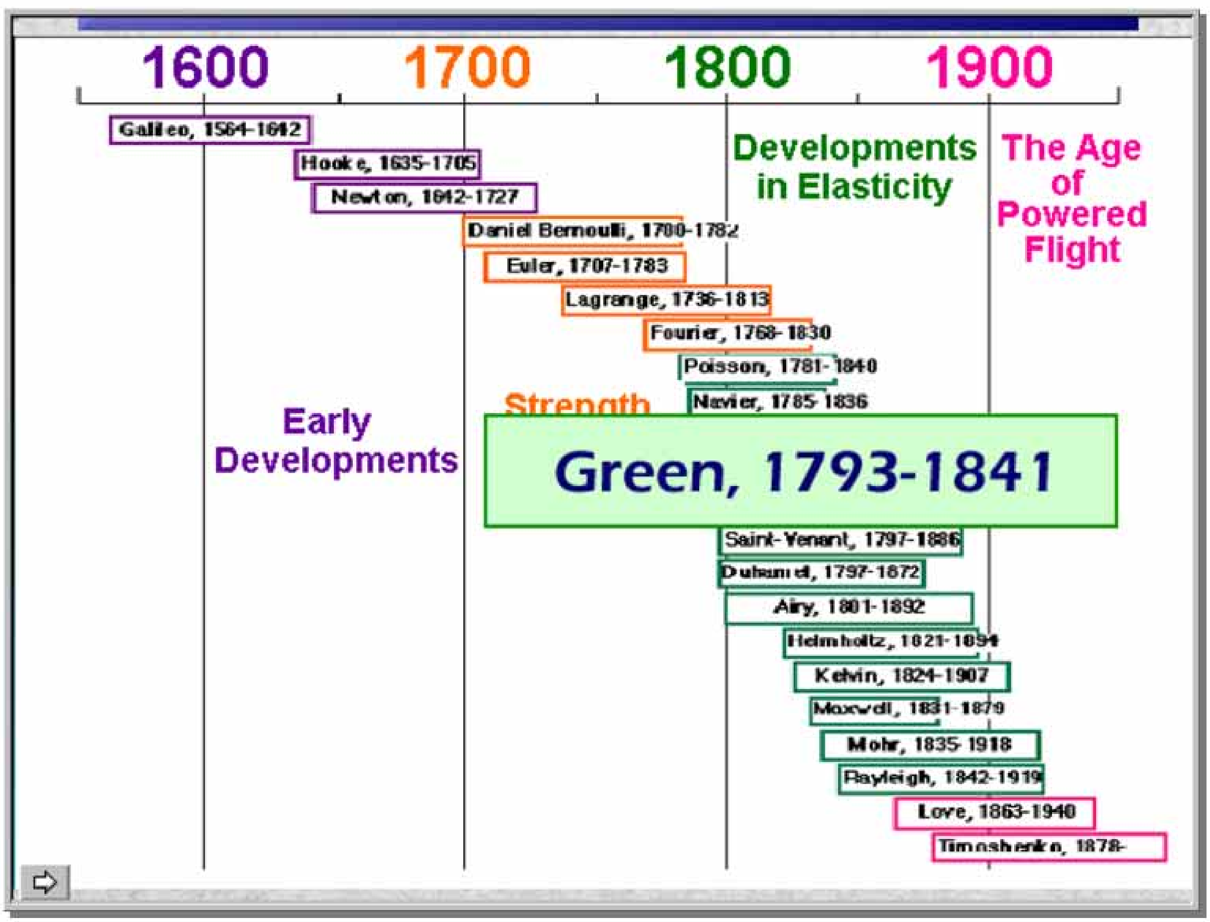
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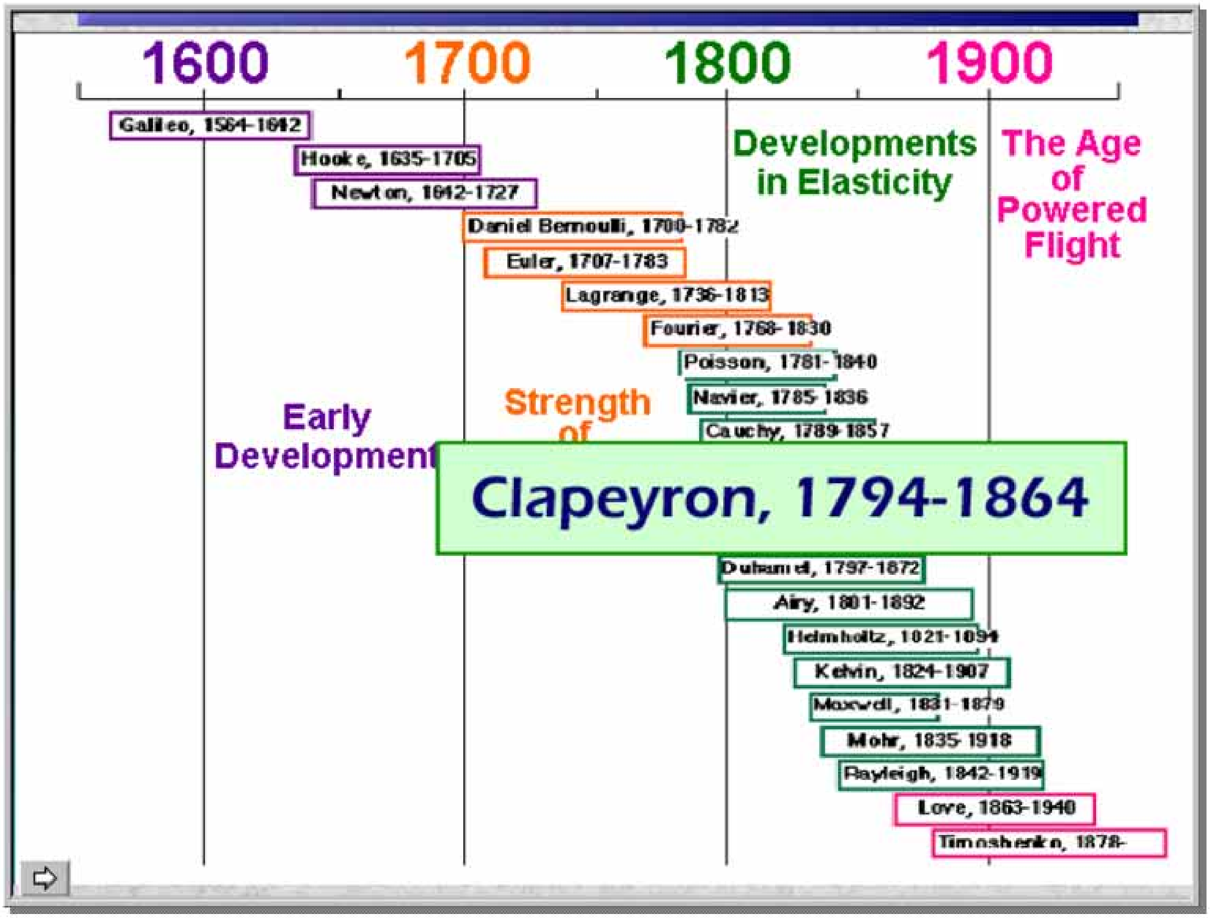
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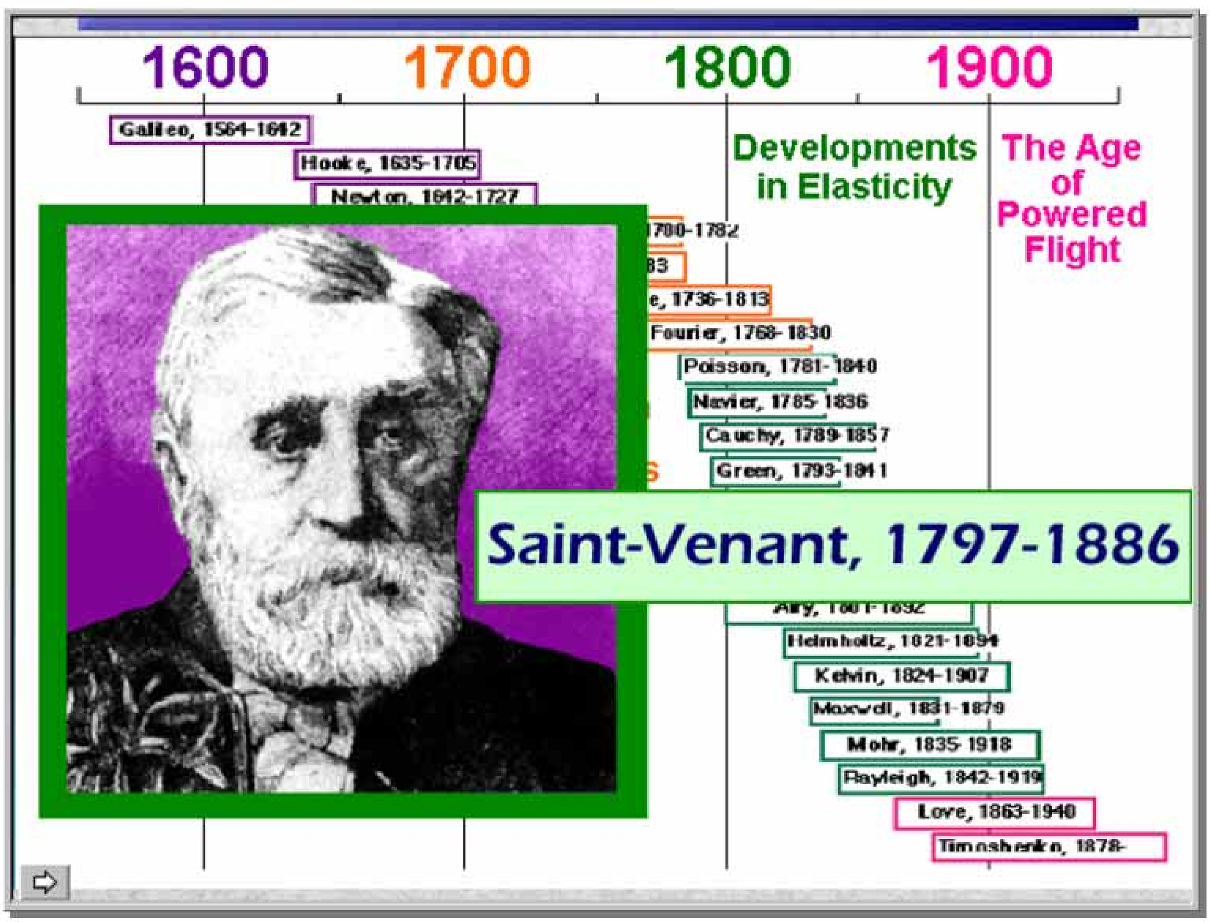
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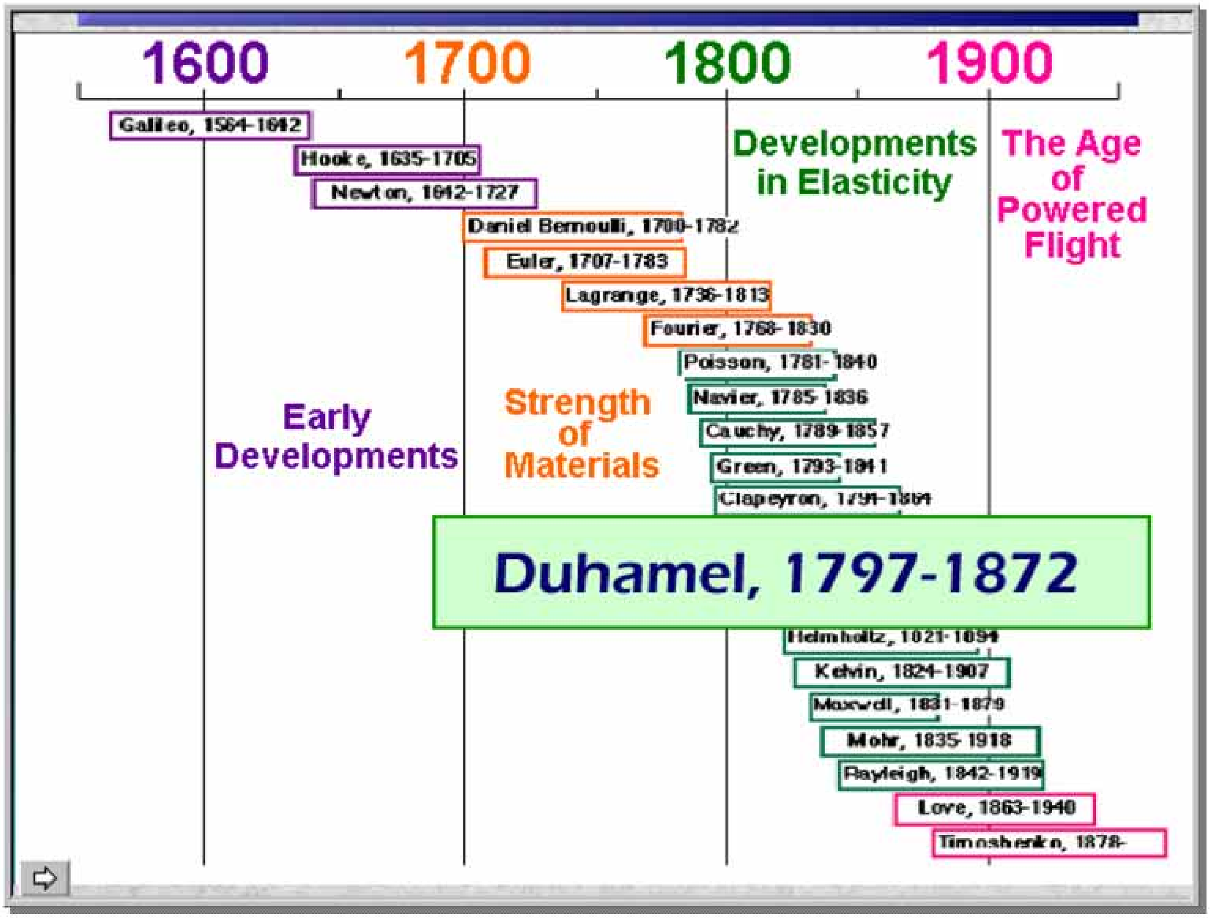

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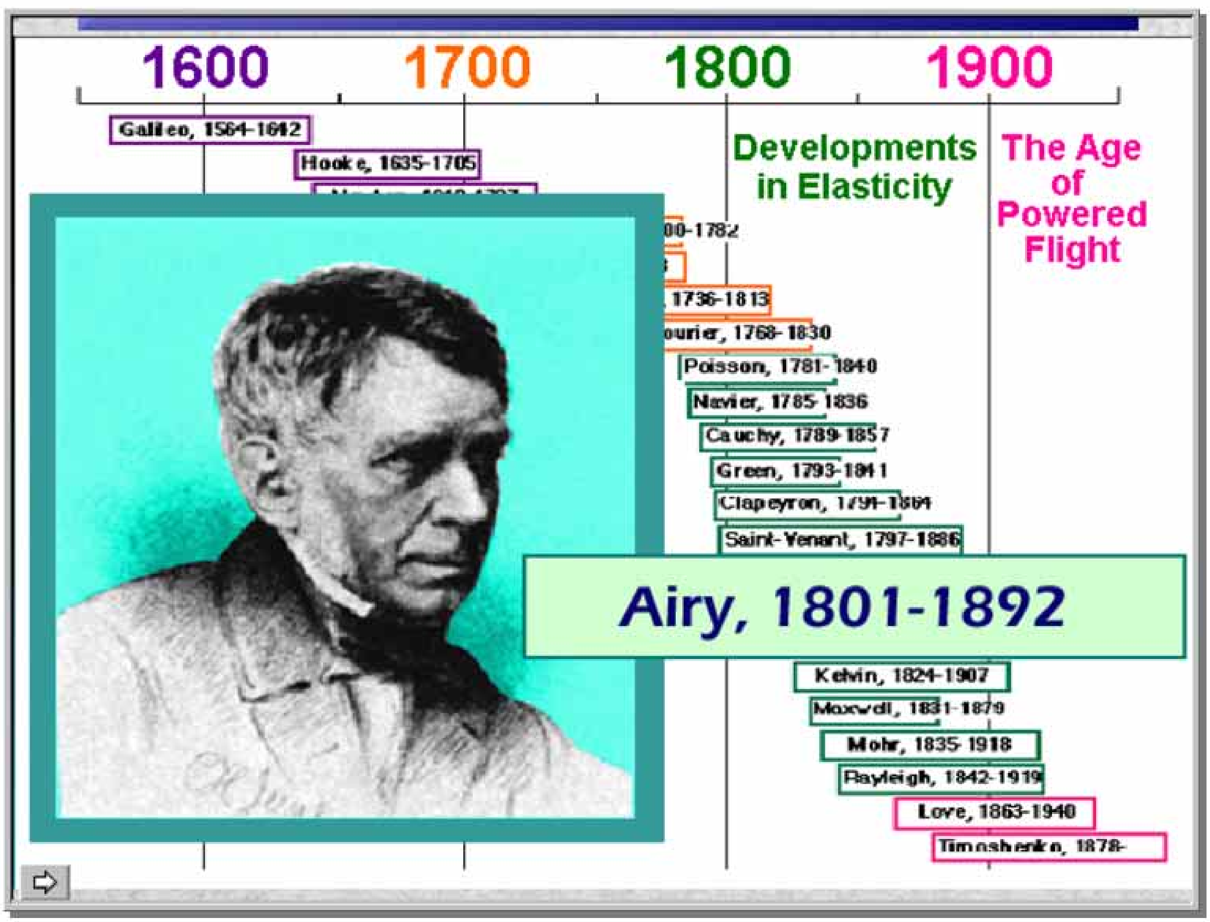

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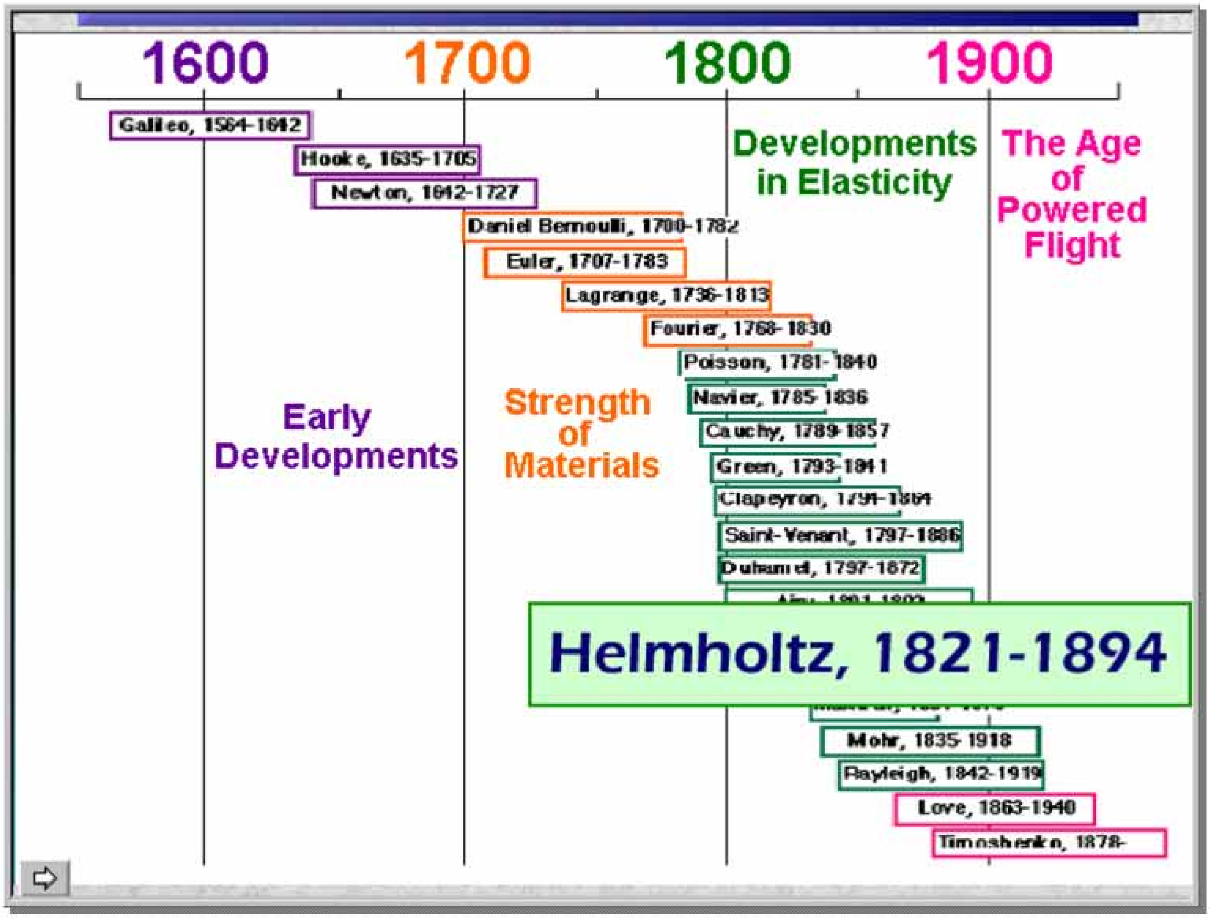
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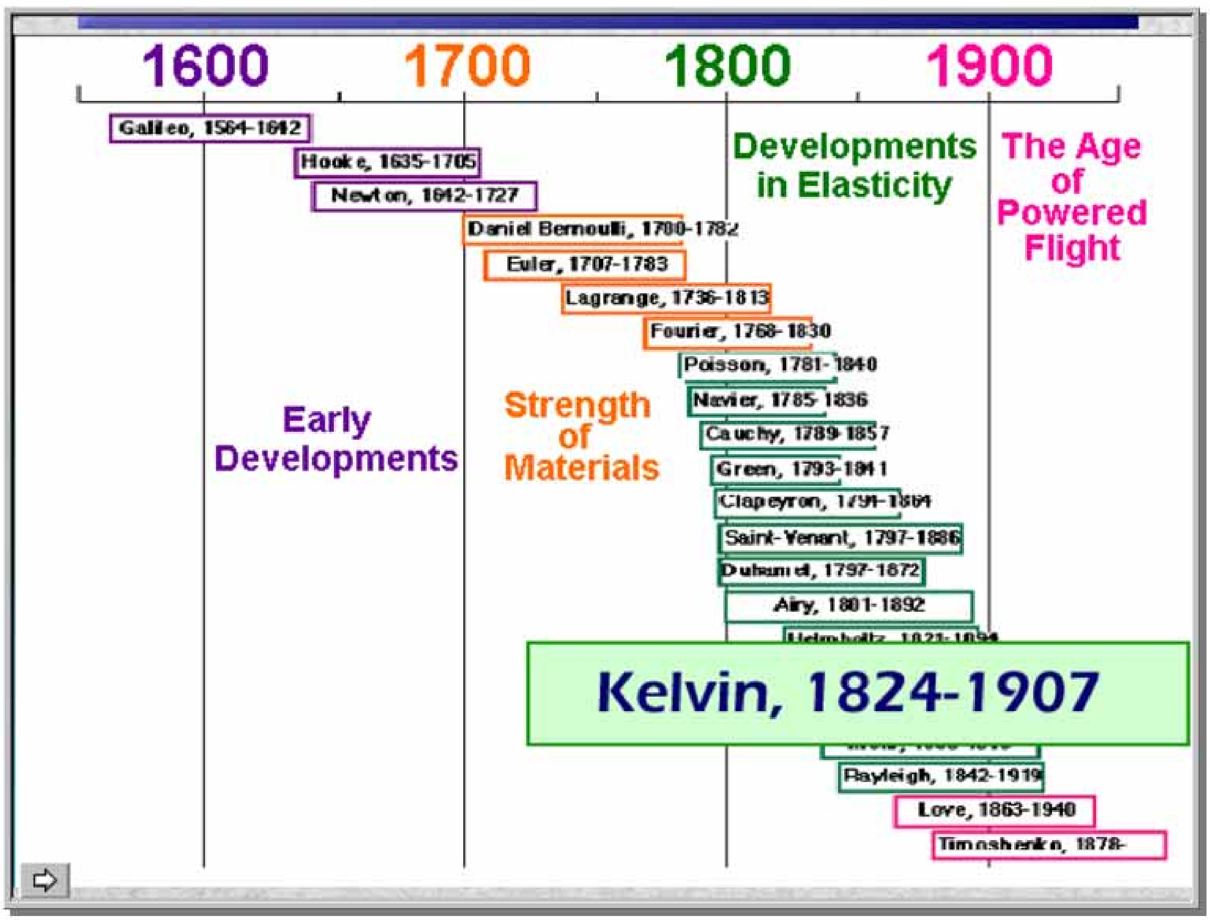
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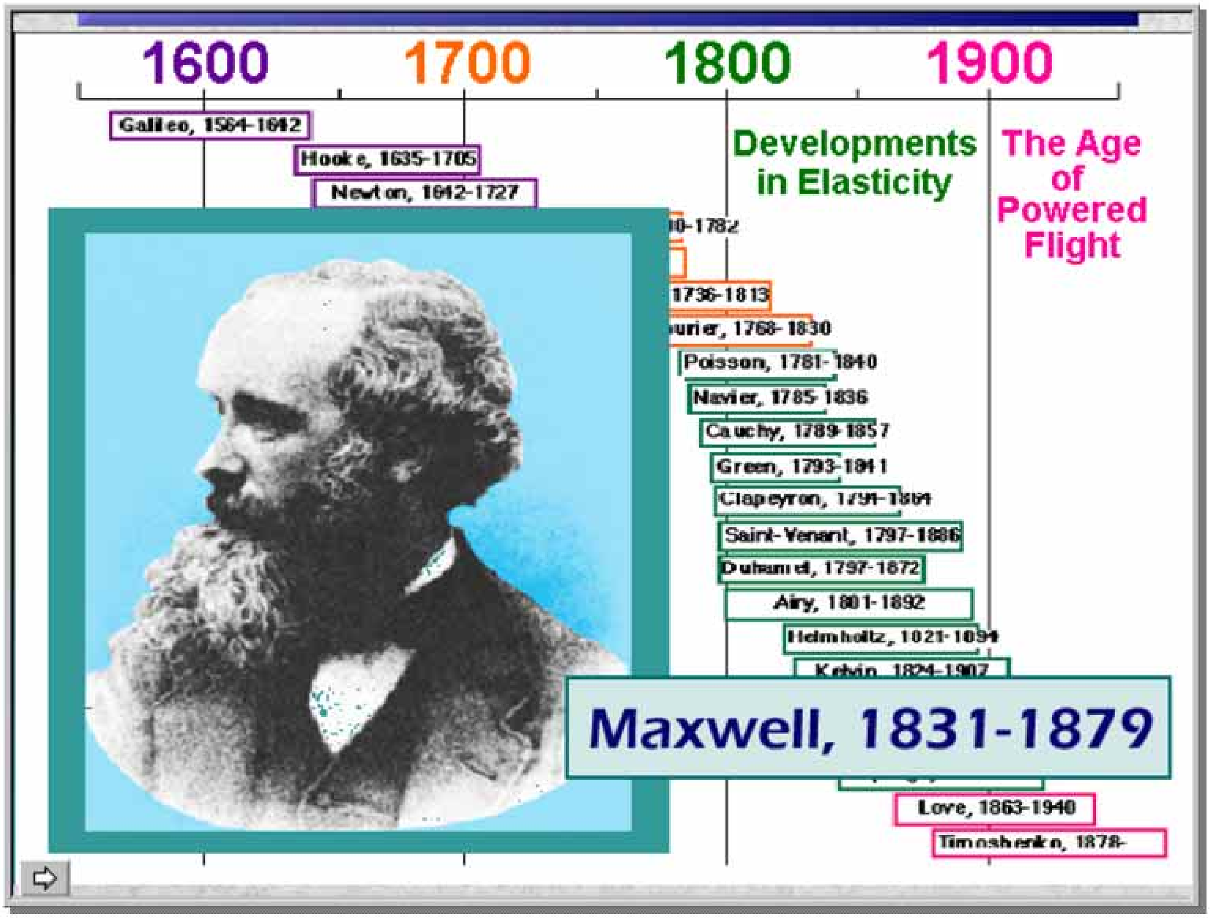
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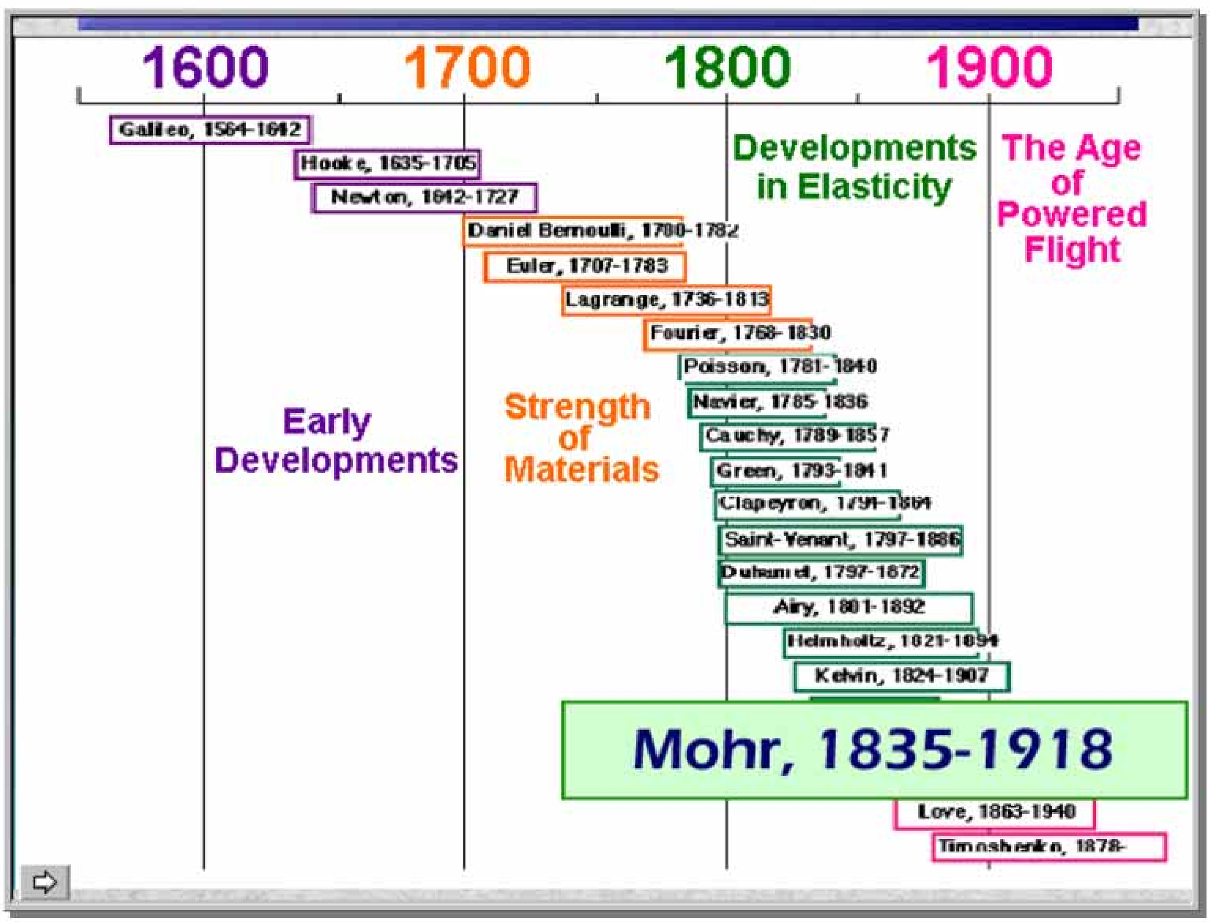
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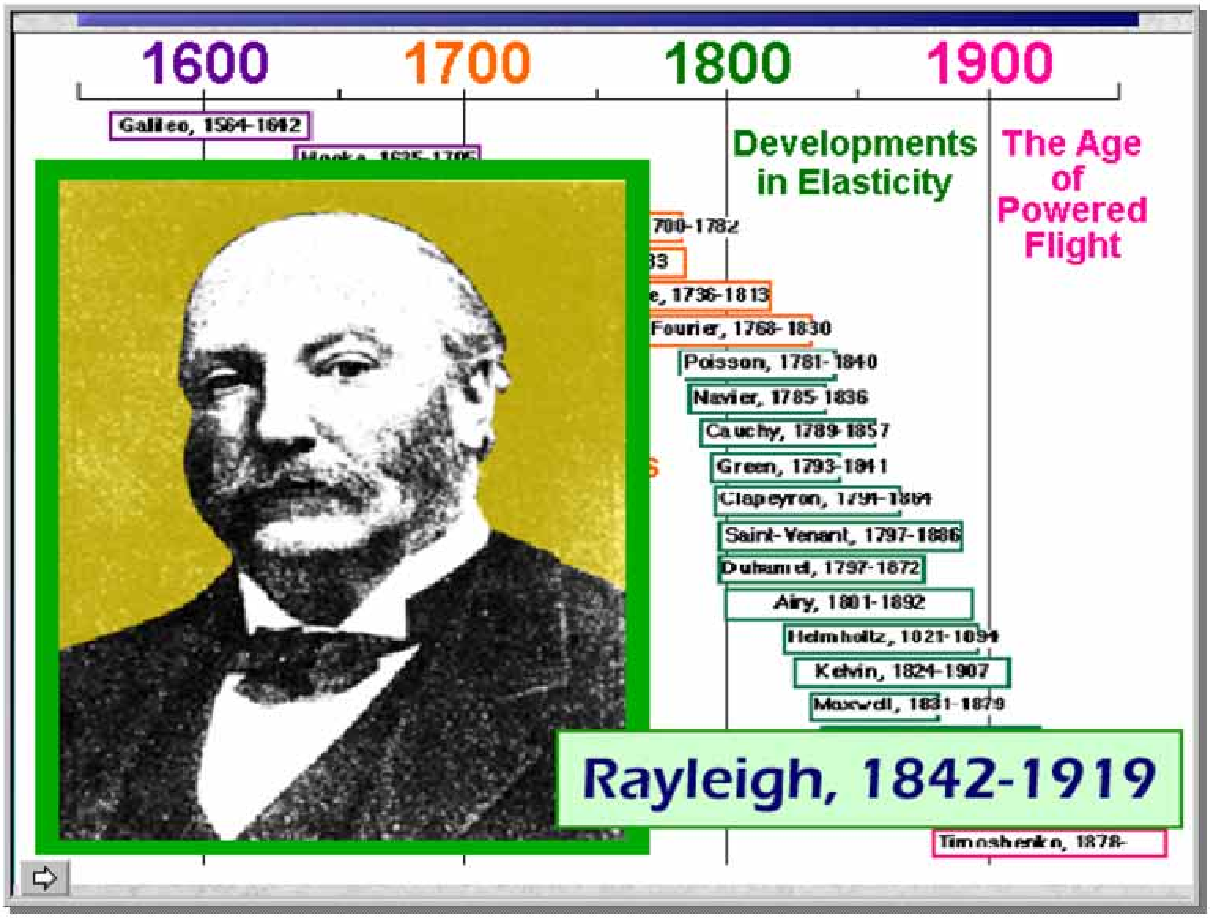

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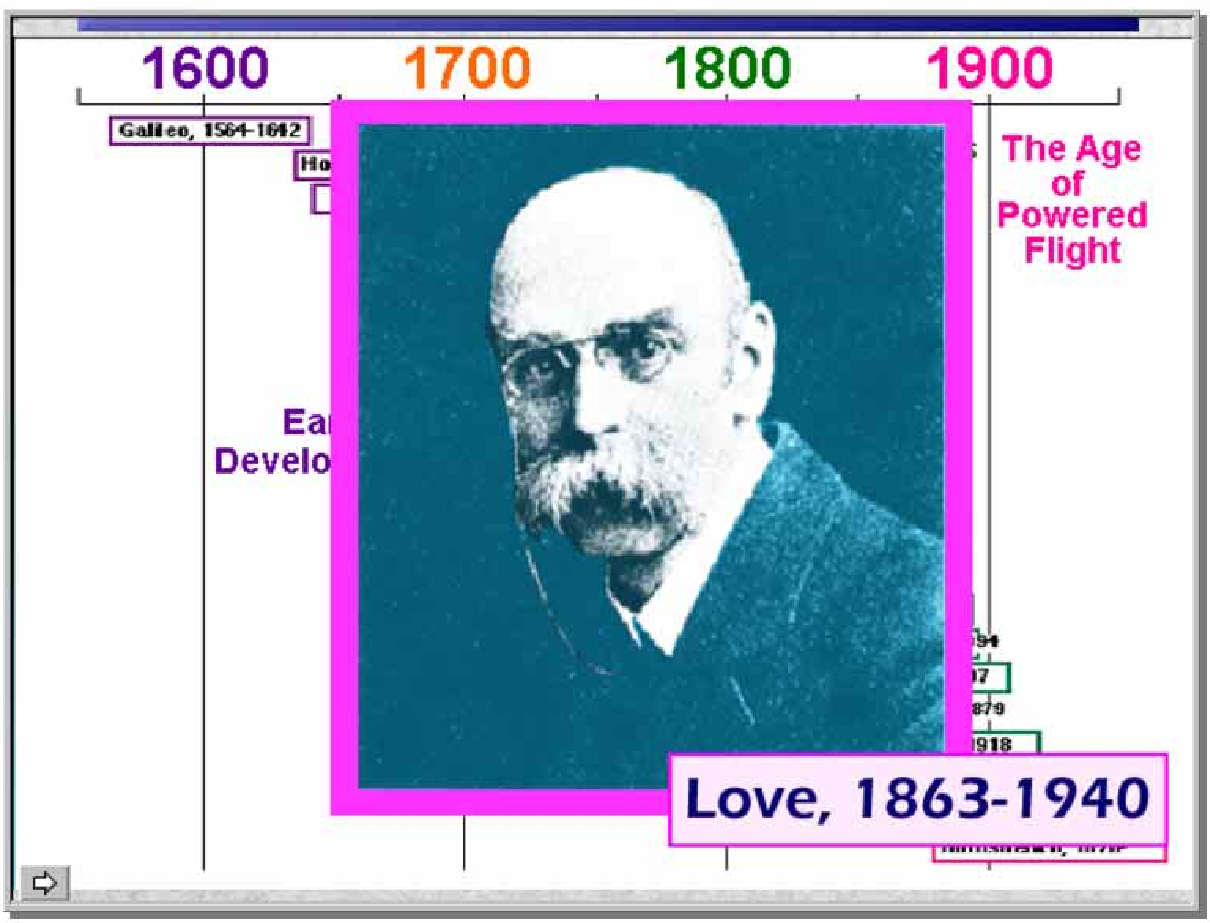

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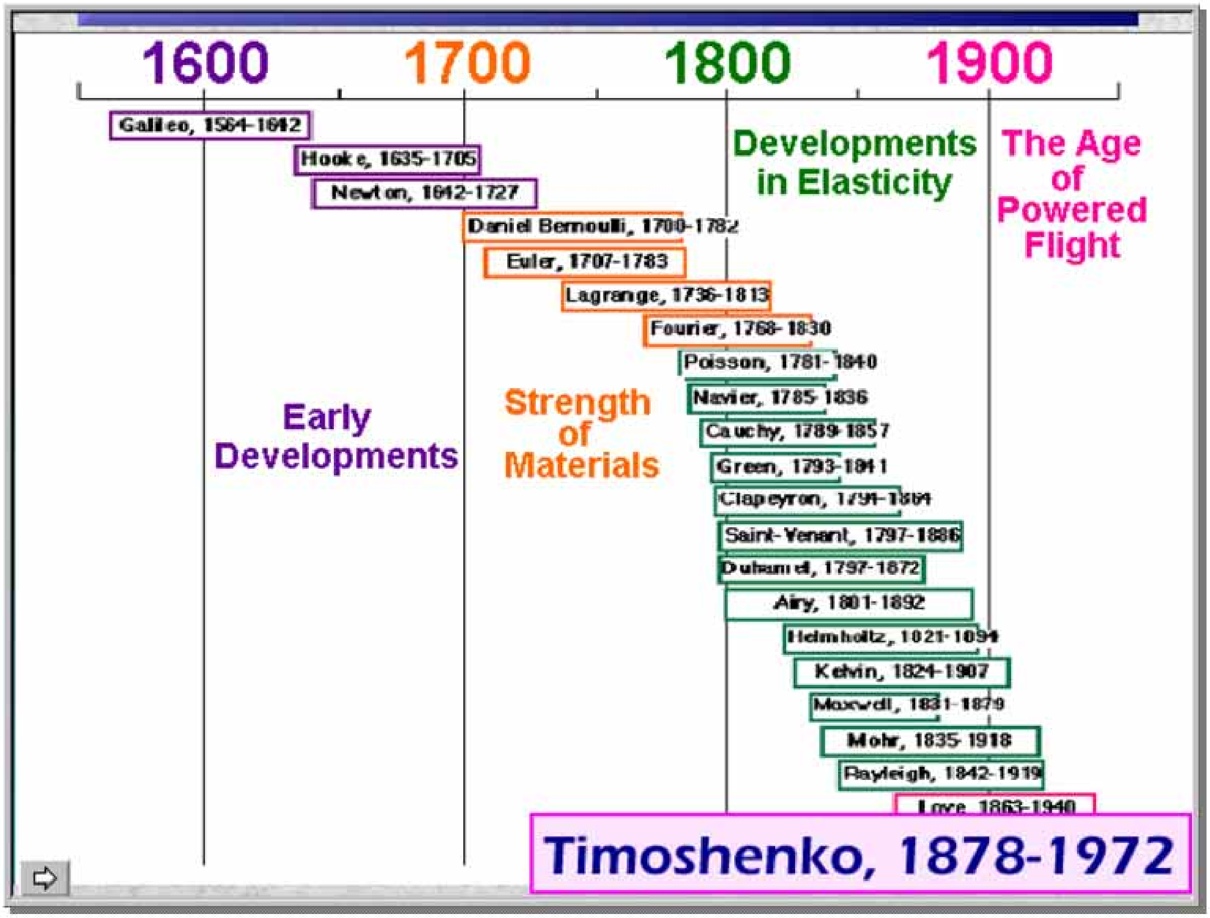


Basic Assumptions in Mechanics of Materials

- Materials - continuous on macroscopic level
- Masses - reasonably large (small masses studied in quantum mechanics)
- Velocities - small compared to speed of light
- Simplifying assumptions usually made on kinematic and kinetic variables and material characteristics

Axioms of Nature

- They are obeyed by all continuous bodies, regardless of their shape or material makeup.
- They cannot be proven rigorously.
- They are rarely, if ever, observed to be violated.

Axioms of Nature

Kinetics

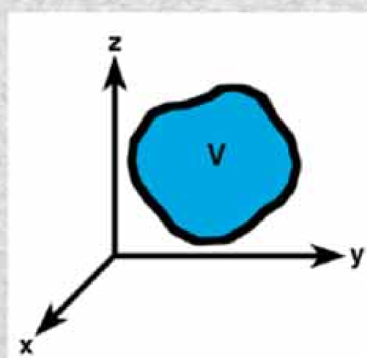
Branch of mechanics dealing with the motions of material bodies under the action of given forces.

Conservation of Mass

$$\frac{dm}{dt} = 0$$

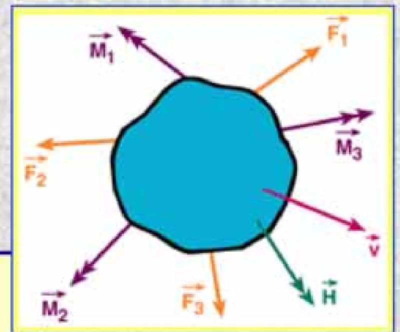
$$m = \int_V \rho \, dV$$

ρ = mass density (mass per unit volume)



Axioms of Nature

Conservation of Momentum



$$\Sigma \vec{F} = \frac{d}{dt}(m\,\vec{v}) \quad , \quad \Sigma \vec{M} = \frac{d}{dt}(\vec{H})$$

$\vec{F} \equiv$ force vectors

$\vec{M} \equiv$ moment vectors

$\vec{v} \equiv$ velocity vector

$\vec{H} \equiv$ angular momentum vector

Axioms of Nature

Thermodynamics

Branch of physics dealing with the conservation of energy from one form to another.

Conservation of Energy

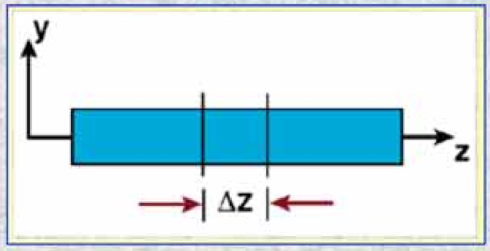
Entropy Production

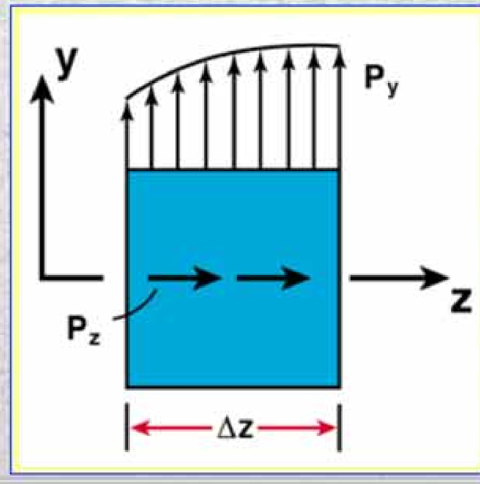
Planar Beams

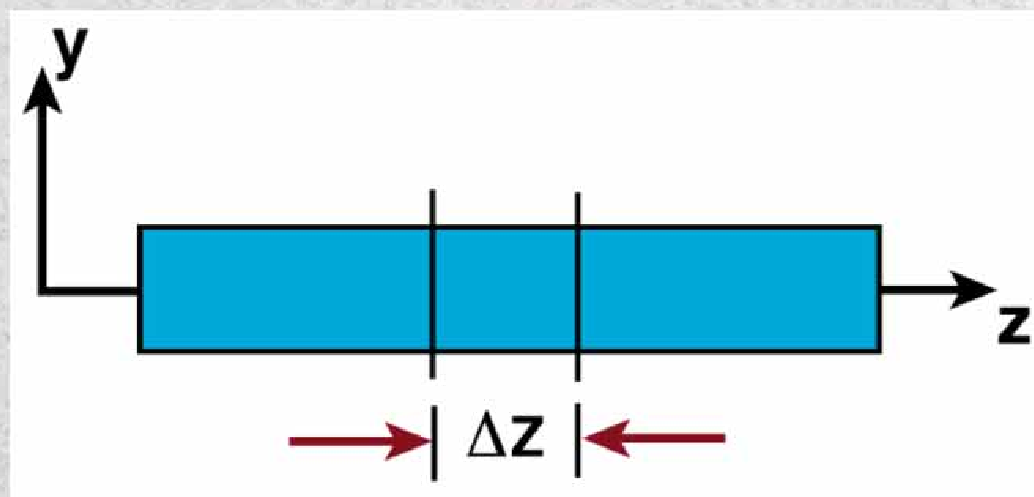
External Loading

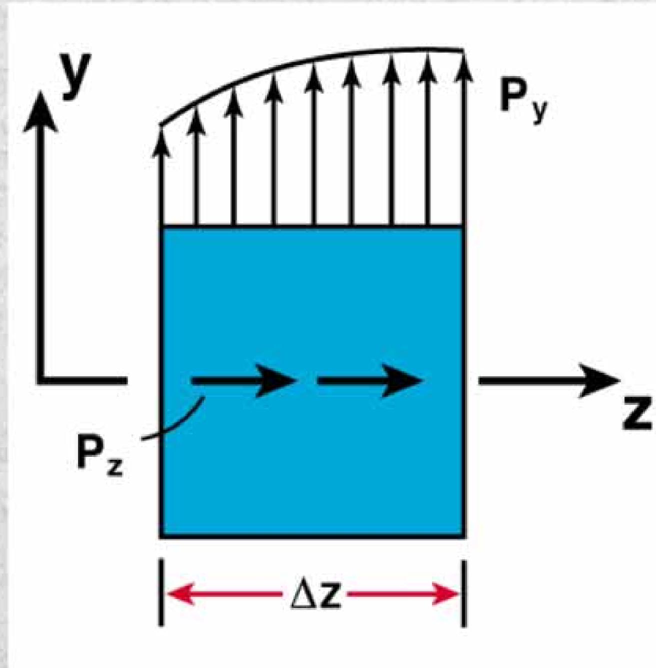
p_y, p_z positive if acting in the positive y and z directions

p_y, p_z intensity of external loadings in the y and z directions







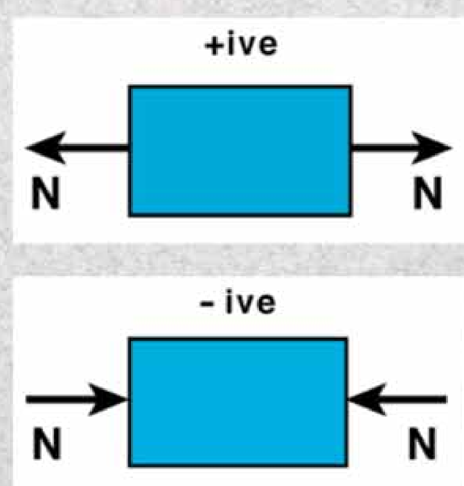


Planar Beams

Internal forces represent resistance to the relative motion of two adjacent cross sections.

Normal Force, N

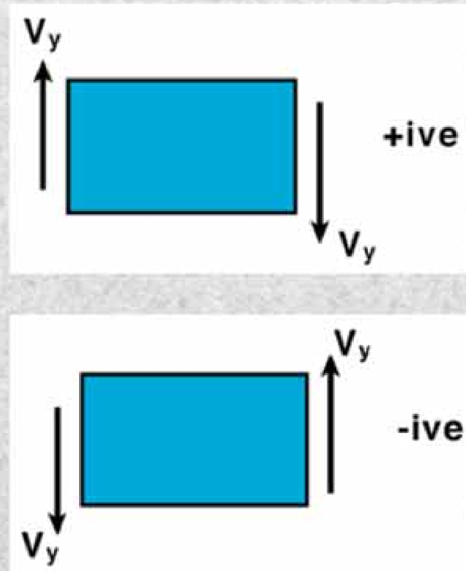
N positive if tensile and negative if compressive



Planar Beams

Shearing Force, V_y

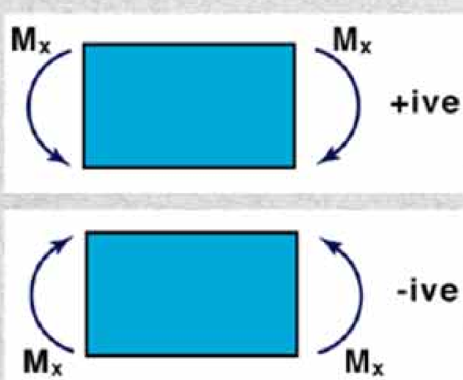
positive (+ive) and negative (-ive) as shown

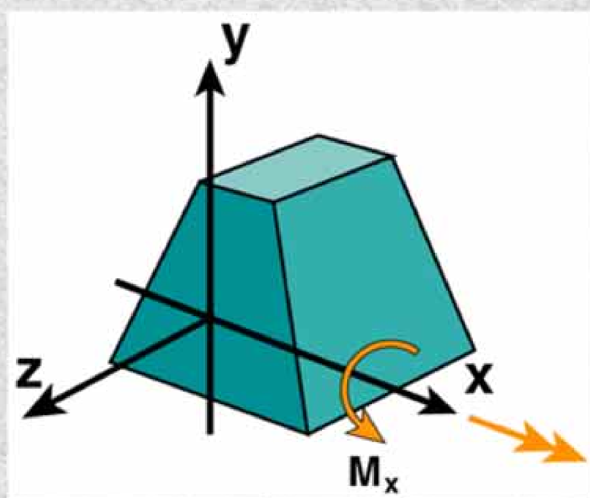


Planar Beams

Bending Moment, M_x

positive (+ive) and negative (-ive) as shown

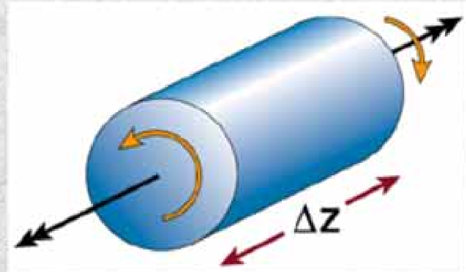




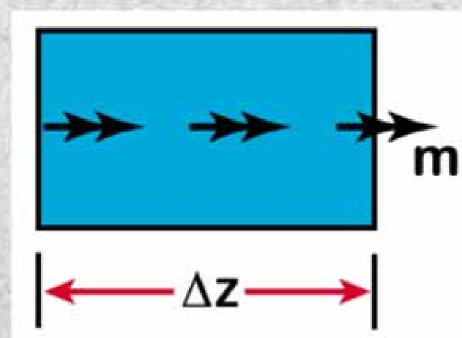
Torsion of Circular Bars

External Twisting Moments

- Right hand screw rule used for representing moments.



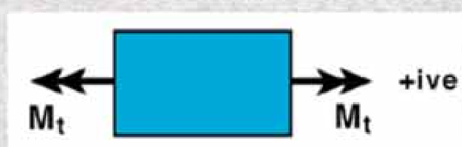
- Intensity of external twisting moment m is positive, if its vector representation is in the positive coordinate direction

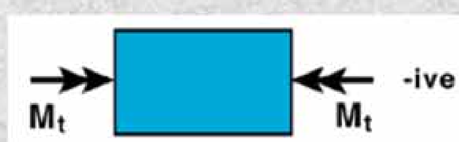


Torsion of Circular Bars

Internal Twisting Moments

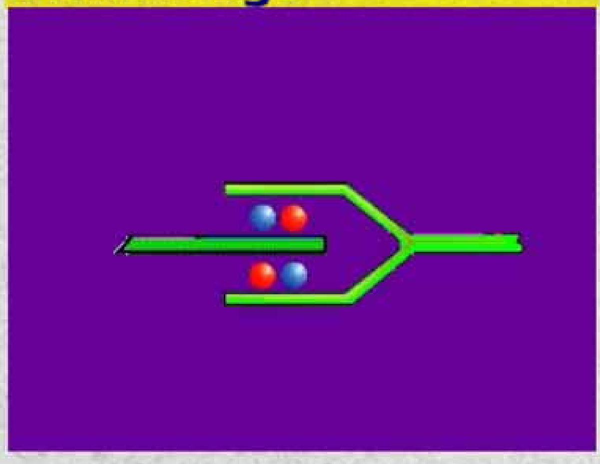
positive (+ive) and negative (-ive) as shown





Intermediate Articulations (Hinges)

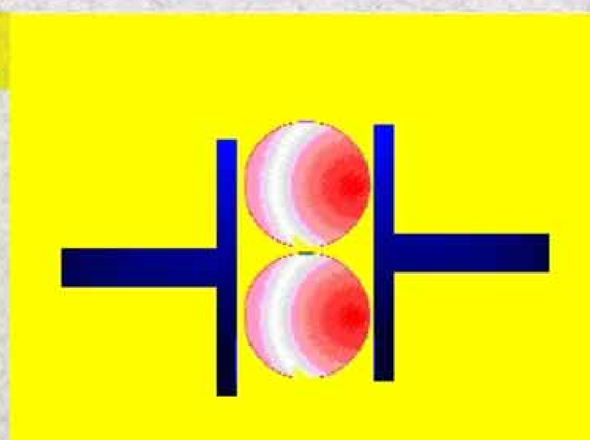
Axial hinge



Intermediate Articulations (Hinges)

Axial hinge

Shear hinge

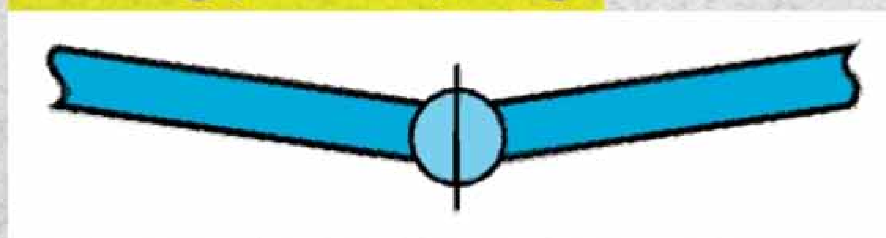


Intermediate Articulations (Hinges)

Axial hinge

Shear hinge

Bending (flexural) hinge

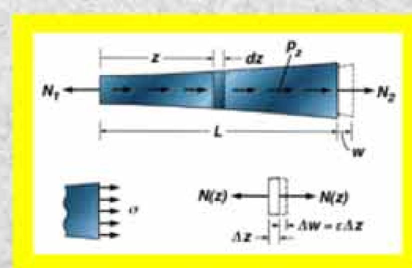


Elementary States of Stress and Strain

1. Axial Loading

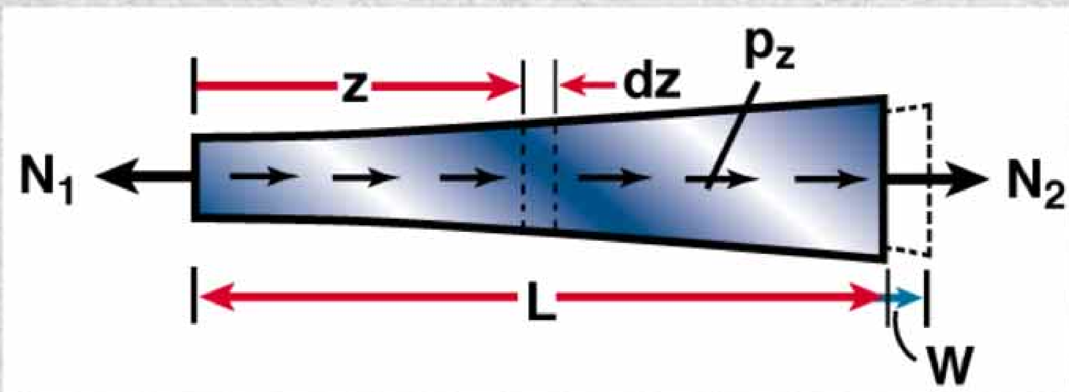
Geometry of Deformation

- Plane cross section remains plane and normal to the axis after deformation.



- Axial strain $\varepsilon = dw/dz$ which is the same at all points of a given cross section.

Elementary States of Stress and Strain



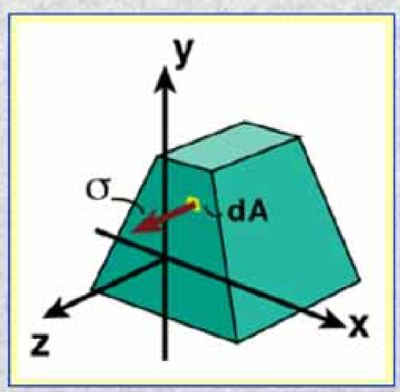
Elementary States of Stress and Strain

1. Axial Loading

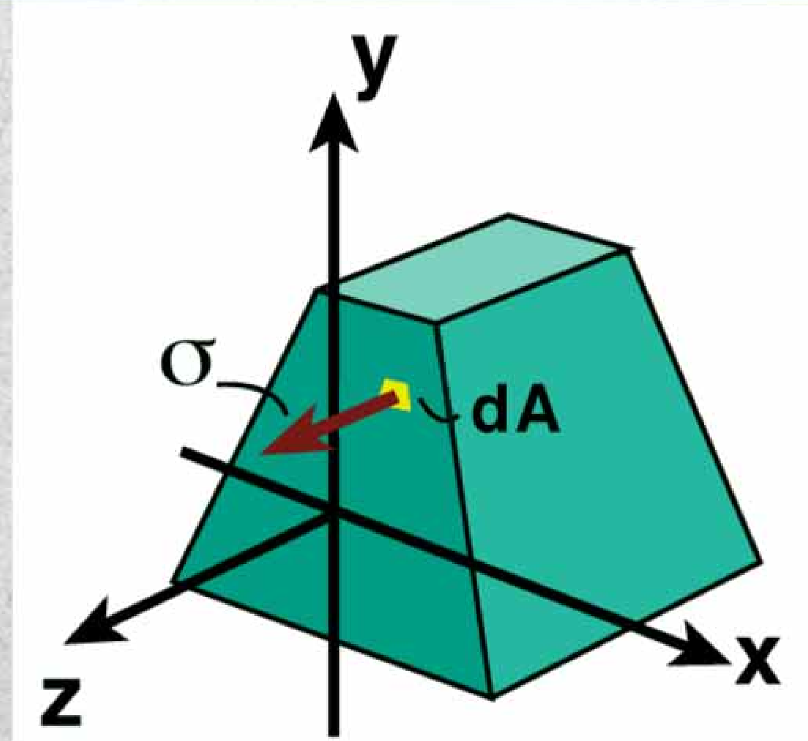
Static Relation - Equilibrium

$$\int_A \sigma \, dA = N$$

where σ = normal stress



Elementary States of Stress and Strain



Elementary States of Stress and Strain

1. Axial Loading

Constitutive Relation

$\sigma = E\,\varepsilon$

E = modulus of elasticity in tension or compression

$$\int_A E\,\varepsilon\, dA = N$$

If E is uniform at all points of the cross section, then

$$E\,\varepsilon \int_A dA = N$$

Elementary States of Stress and Strain

1. Axial Loading

Constitutive Relation

$$\int_A E\,\varepsilon\,dA = N$$

If E is uniform at all points of the cross section, then

$$E\,\varepsilon \int_A dA = N$$

or

$$E\,\varepsilon\,A = N$$

Elementary States of Stress and Strain

1. Axial Loading

Constitutive Relation

$$E\,\varepsilon \int_A dA = N$$

or $E\,\varepsilon\,A = N$

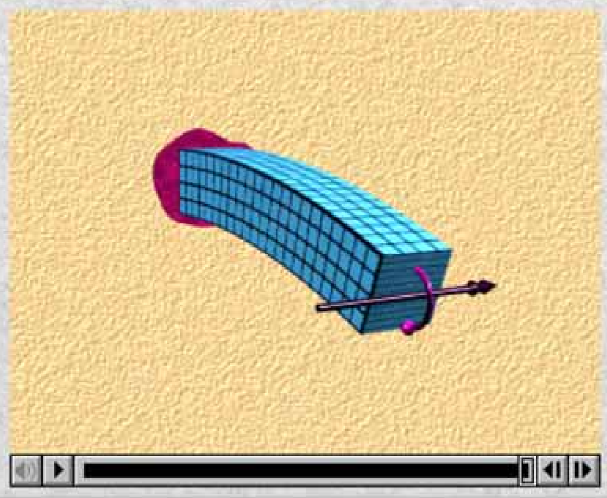
$\sigma = N/A$

$\varepsilon = \dfrac{dw}{dz} = \dfrac{N}{EA}$ and EA = extensional stiffness

Elementary States of Stress and Strain

2. Pure and Transverse Bending

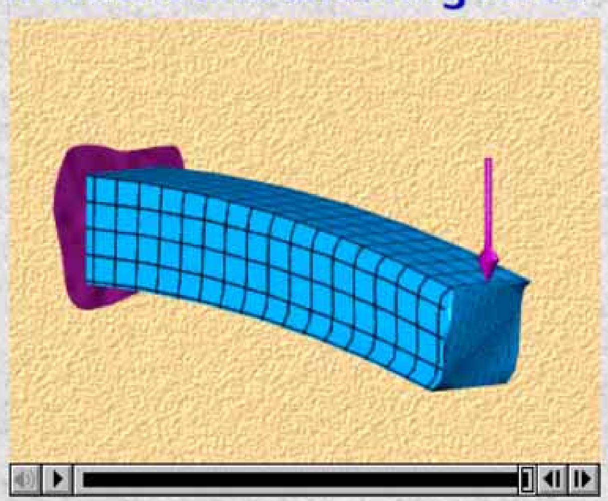
Pure bending refers to bending moment only (no axial force, shear force, or twisting moment)



Elementary States of Stress and Strain

2. Pure and Transverse Bending

Transverse bending refers to combination of bending moment and shearing force

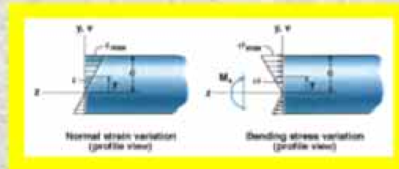


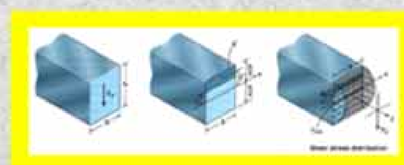
Elementary States of Stress and Strain

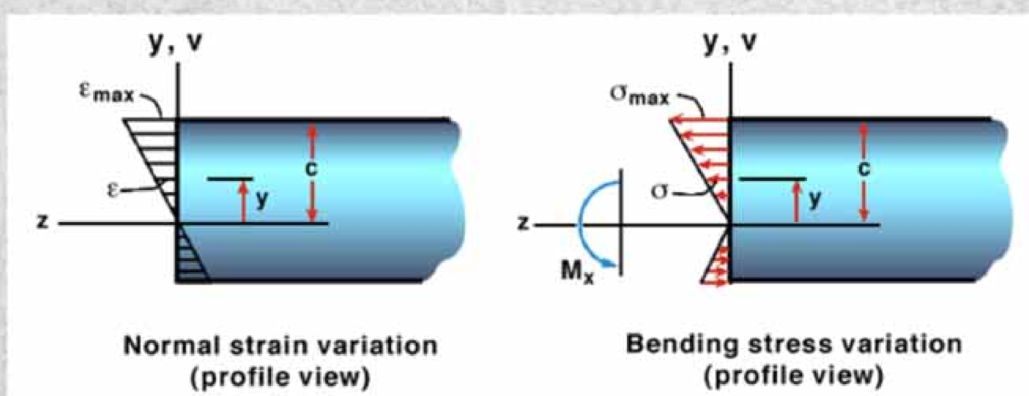
2. Pure and Transverse Bending

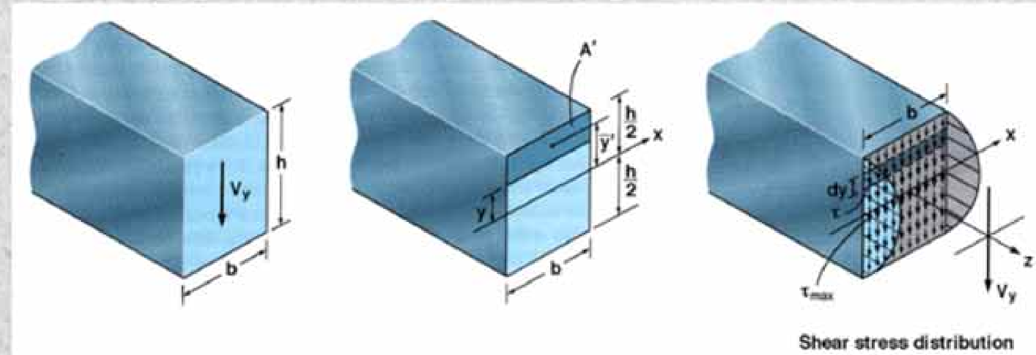
Geometry of Deformation

- Plane cross section before bending remains plane after bending and normal to the center line of the beam.
- The neutral axis is the x-axis.







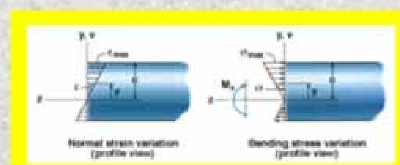


Elementary States of Stress and Strain

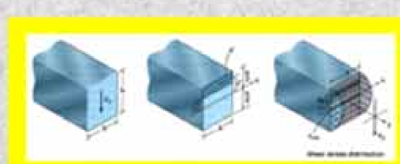
2. Pure and Transverse Bending

Geometry of Deformation

$$\kappa = -\frac{d^2v}{dz^2}$$



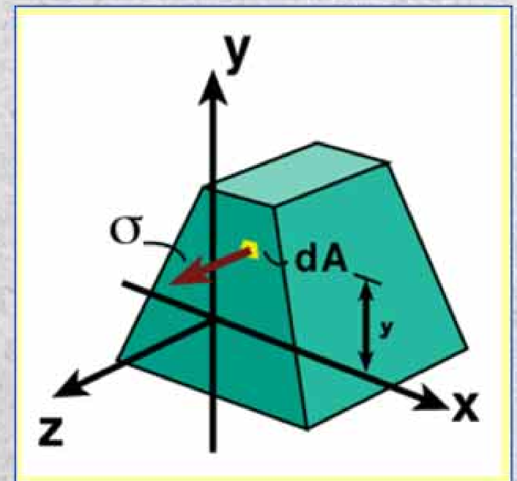
$$\varepsilon = y \times \kappa = -y\,\frac{d^2v}{dz^2}$$



Elementary States of Stress and Strain

2. Pure and Transverse Bending

Static Relation - Equilibrium



$$\int_A \sigma \, y \, dA = M_x$$

Elementary States of Stress and Strain

2. Pure and Transverse Bending

Constitutive Relation

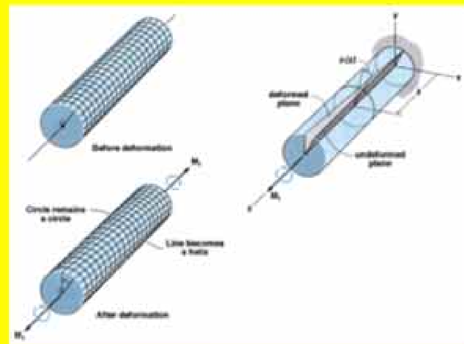
$$\sigma = E\,\varepsilon$$
$$= E\,y\,\kappa$$
$$M_x = E\,I_x\,\kappa = -E\,I_x\,\frac{d^2v}{dz^2}$$
$$\sigma = \frac{M_x}{I_x}\,y$$

Elementary States of Stress and Strain

3. Torsion of Bars with Circular Cross Section

Geometry of Deformation

- Plane parallel cross sections remain plane and parallel after deformation.



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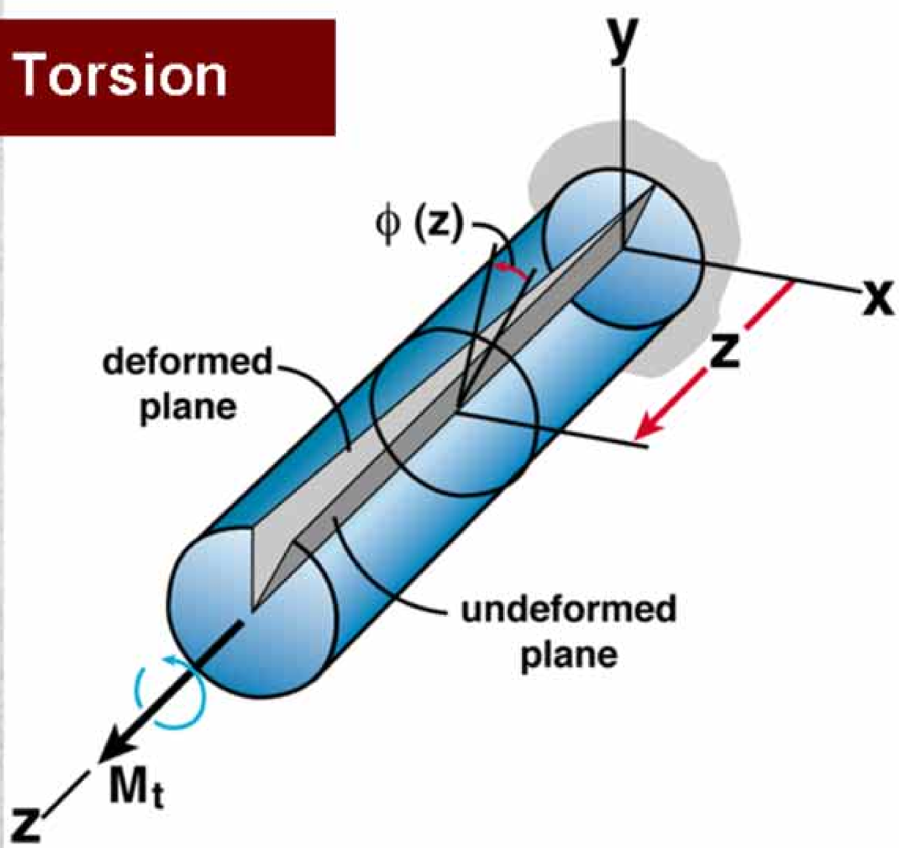
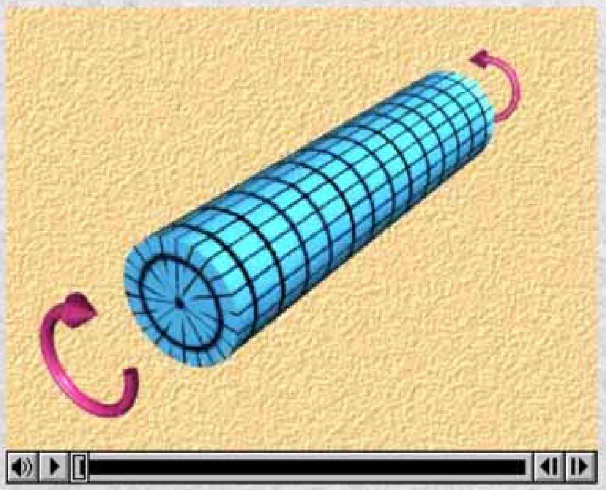
Torsion





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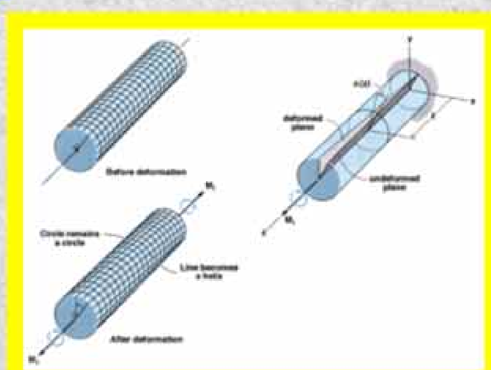
Elementary States of Stress and Strain

3. Torsion of Bars with Circular Cross Section

Geometry of Deformation

- Diameters of cross sections and distances between them do not change.



animation

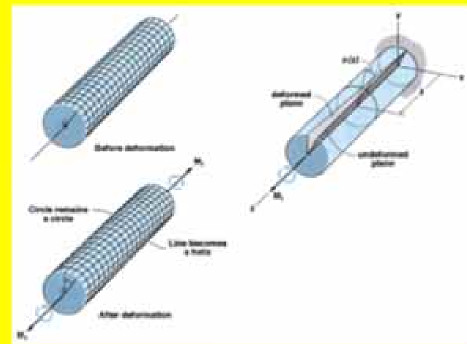
Elementary States of Stress and Strain

3. Torsion of Bars with Circular Cross Section

Geometry of Deformation

Shearing strain

$$\gamma = r\frac{d\varphi}{dz}$$



animation

Elementary States of Stress and Strain

3. Torsion of Bars with Circular Cross Section

Static Relation - Equilibrium

$$\int_A \tau \, r \, dA = M_t$$

Elementary States of Stress and Strain

3. Torsion of Bars with Circular Cross Section

Constitutive Relation

$$\tau = G\,\gamma$$

$$M_t = G\,I_p\,\frac{d\varphi}{dz}\ ,\quad \left(I_p = \int_A r^2\,dA\right)$$

$$\tau = \frac{M_t}{I_p}\,r$$

Elementary States of Stress and Strain

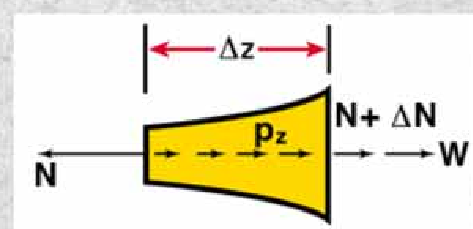
4. Relations between External and Internal Forces

Force Equilibrium

Axial Forces

$$p_z\,\Delta z + \Delta N = 0$$

$$\frac{dN}{dz} = -\,p_z$$



Elementary States of Stress and Strain

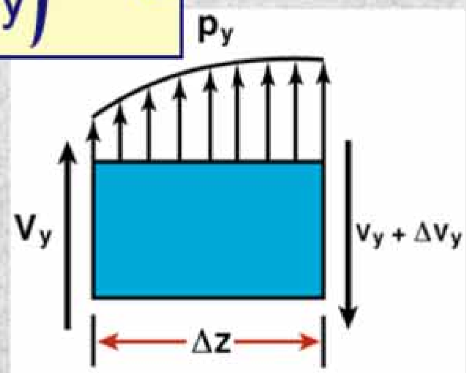
4. Relations between External and Internal Forces

Force Equilibrium

Transverse Forces

$$p_y \Delta z + V_y - \left(V_y + \Delta V_y\right) = 0$$

$$\frac{dV_y}{dz} = p_y$$

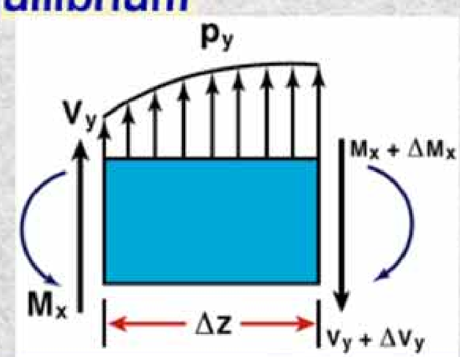


Elementary States of Stress and Strain

4. Relations between External and Internal Forces

Moment Equilibrium

Bending Moments



$$v_y \Delta z + \left(M_x + \Delta M_x\right) - M_x - p_y \frac{\left(\Delta z\right)^2}{2} = 0$$

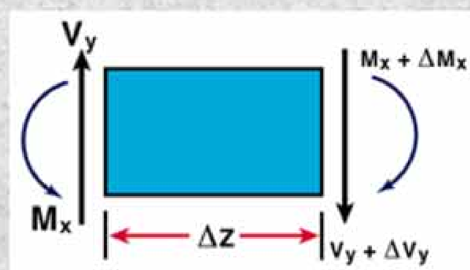
Elementary States of Stress and Strain

4. Relations between External and Internal Forces

Moment Equilibrium

Bending Moments

$$\frac{dM_x}{dz} = -V_y$$



Elementary States of Stress and Strain

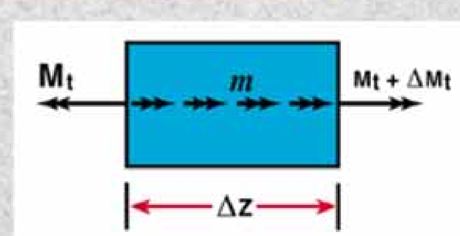
4. Relations between External and Internal Forces

Moment Equilibrium

Twisting Moments

$$\Delta z\, m + \left(M_t + \Delta M_t\right) - M_t = 0$$

$$\frac{dM_x}{dz} = -m$$

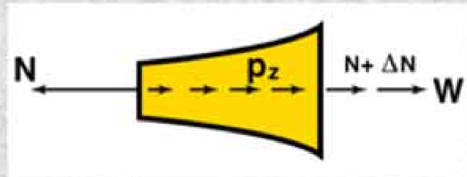


Elementary States of Stress and Strain

5. Governing Equations

Axial Loading

$$\boxed{\frac{dN}{dz} = -p_z}$$



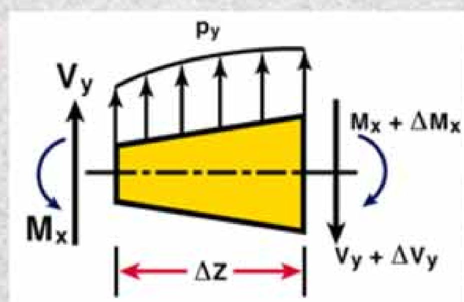
$$\boxed{N = EA\frac{dw}{dz}}$$

$$\boxed{\frac{d}{dz}\left(EA\frac{dw}{dz}\right) = -p_z}$$

Elementary States of Stress and Strain

5. Governing Equations

Pure and Transverse Bending



$$\boxed{\frac{dV_y}{dz} = p_y}$$

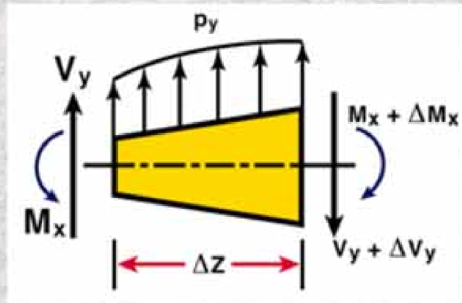
$$\boxed{\frac{dM_x}{dz} = -V_y}$$

Elementary States of Stress and Strain

5. Governing Equations

Pure and Transverse Bending

$$\frac{d^2M_x}{dz^2} = -p_y$$

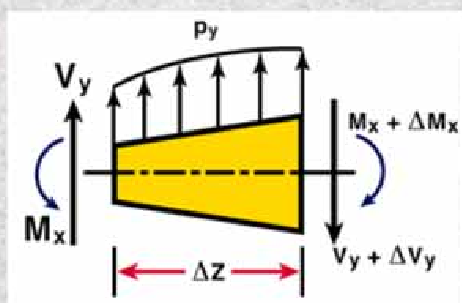


Elementary States of Stress and Strain

5. Governing Equations

Pure and Transverse Bending

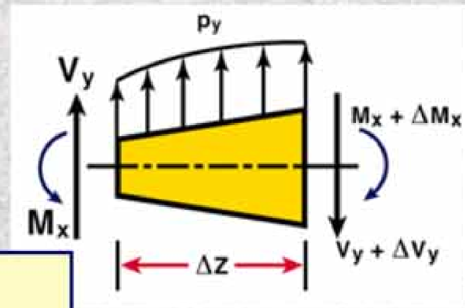
$$M_x = -E\, I_x \frac{d^2v}{dz^2}$$



Elementary States of Stress and Strain

5. Governing Equations

Pure and Transverse Bending



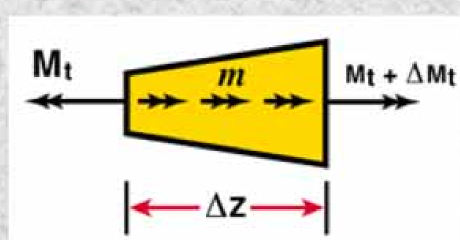
$$\frac{d^2}{dz^2}\left(E\, I_x \frac{d^2 v}{dz^2}\right) = p_y$$

Elementary States of Stress and Strain

5. Governing Equations

Torsion

$$\frac{dM_t}{dz} = -m$$

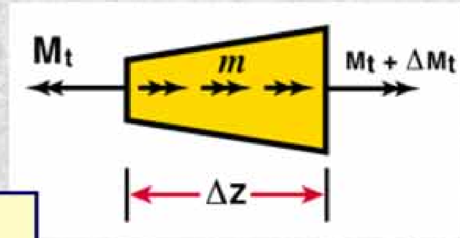


$$M_t = G\, I_p \frac{d\phi}{dz}$$

Elementary States of Stress and Strain

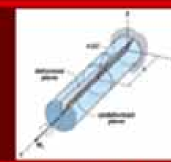
5. Governing Equations

Torsion

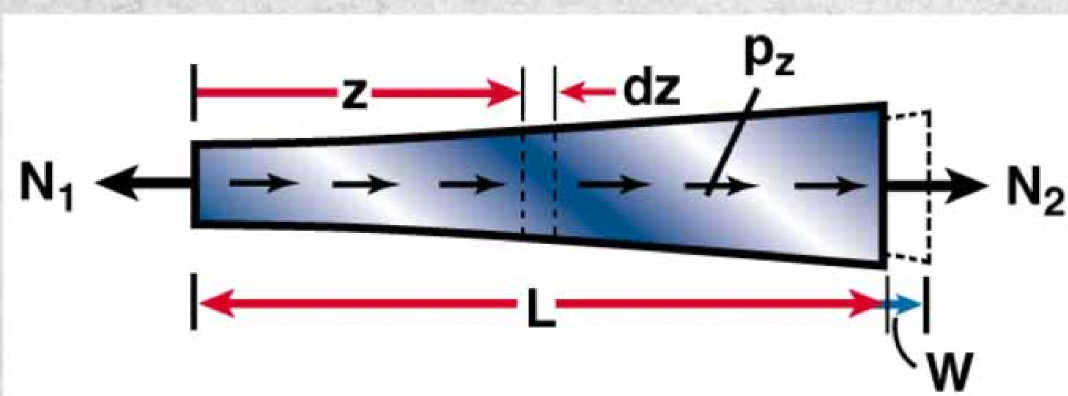


$$\frac{d}{dz}\left(G\, I_p \frac{d\phi}{dz}\right) = -m$$

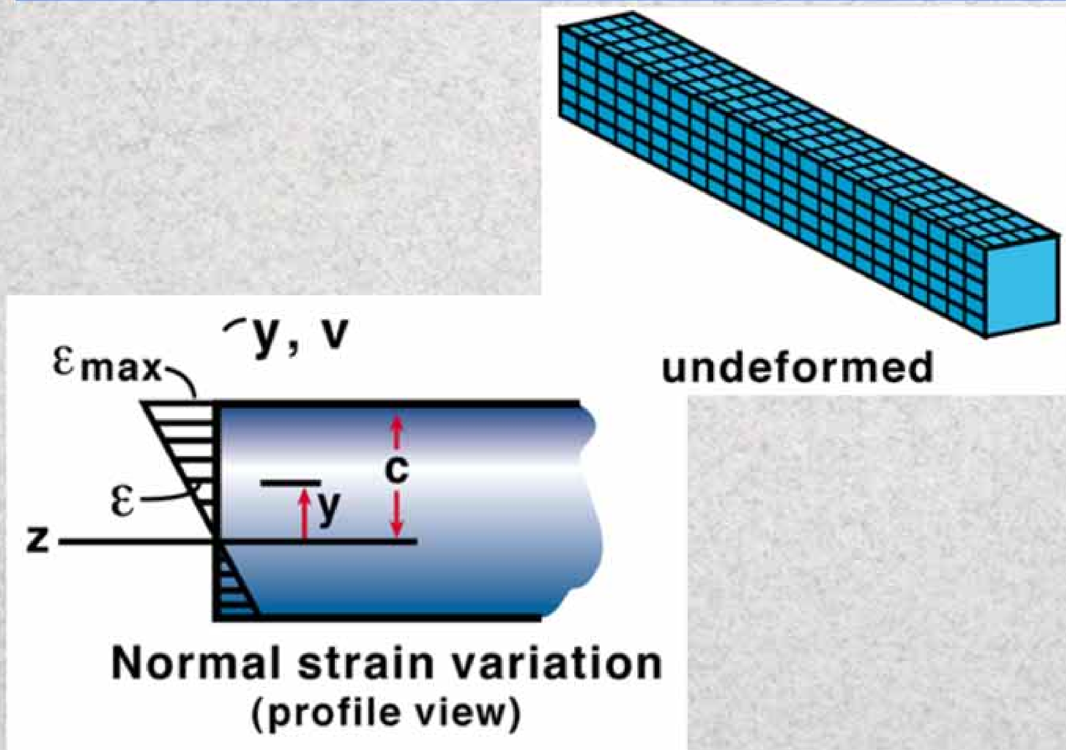
Elementary States of Stress and Strain (Summary)

		Axial Loading	Pure and Transverse Bending	Torsion
Example			 	
Kinematic Relations	Basic Assumption	Plane cross section before deformation remains plane after deformation		
	Strain Displacement Relations	$\varepsilon = \frac{dw}{dz}$	$\kappa \cong -\frac{d^2v}{dz^2}$ $\varepsilon = -y\,\frac{d^2v}{dz^2}$	$\gamma = r\,\frac{d\varphi}{dz}$
Static Relations		$\frac{dN}{dz} = -p_z$	$\frac{dM_x}{dz} = V_y$ $\frac{dV_y}{dz} = -p_y$	$\frac{dM_t}{dz} = -m$
Constitutive Relations		$\sigma = E\,\varepsilon$ $N = EA\,\varepsilon$	$\sigma = E\,\varepsilon = \frac{M_x\,y}{I_x}$ $M_x = E\,I_x\,\kappa$	$\tau = G\,\gamma = \frac{M_t\,r}{I_p}$ $M_t = G\,I_p\,\frac{d\varphi}{dz}$

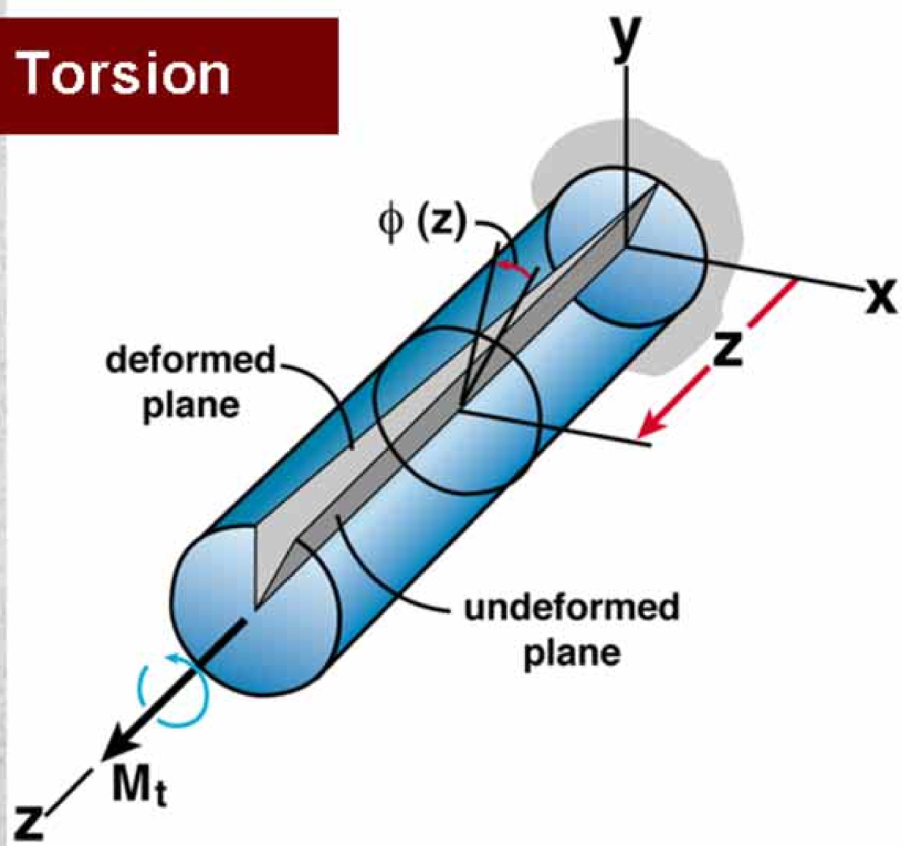
Axial Loading



Pure and Transverse Bending



Torsion



Axial Loading

Strain Displacement Relations

$$\varepsilon = \frac{dw}{dz}$$

Pure and Transverse Bending

Strain Displacement Relations

$$\kappa \cong -\frac{d^2v}{dz^2}$$

$$\varepsilon = -y\,\frac{d^2v}{dz^2}$$

Torsion

Strain Displacement Relations

$$\gamma = r\,\frac{d\varphi}{dz}$$

Axial Loading

Static Relations

$$\frac{dN}{dz} = -p_z$$

Pure and Transverse Bending

Static Relations

$$\frac{dM_x}{dz} = V_y$$

$$\frac{dV_y}{dz} = -p_y$$

Torsion

Static Relations

$$\frac{dM_t}{dz} = -m$$

Axial Loading

Constitutive Relations

$$\sigma = E\,\varepsilon$$
$$N = EA\,\varepsilon$$

Pure and Transverse Bending

Constitutive Relations

$$\sigma = E\,\varepsilon = \frac{M_x\,y}{I_x}$$

$$M_x = E\,I_x\,\kappa$$

Torsion

Constitutive Relations

$$\tau = G\,\gamma = \frac{M_t\, r}{I_p}$$

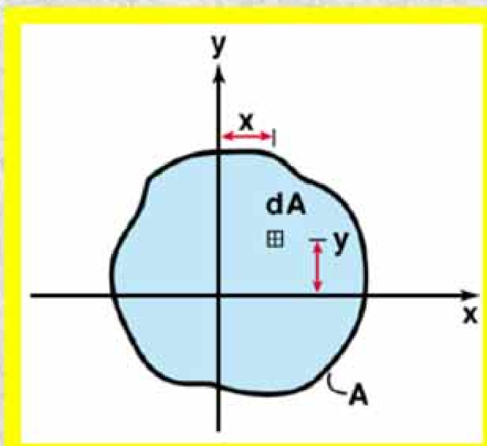
$$M_t = G\, I_p\, \frac{d\varphi}{dz}$$

Geometric Properties of Plane Cross Sections

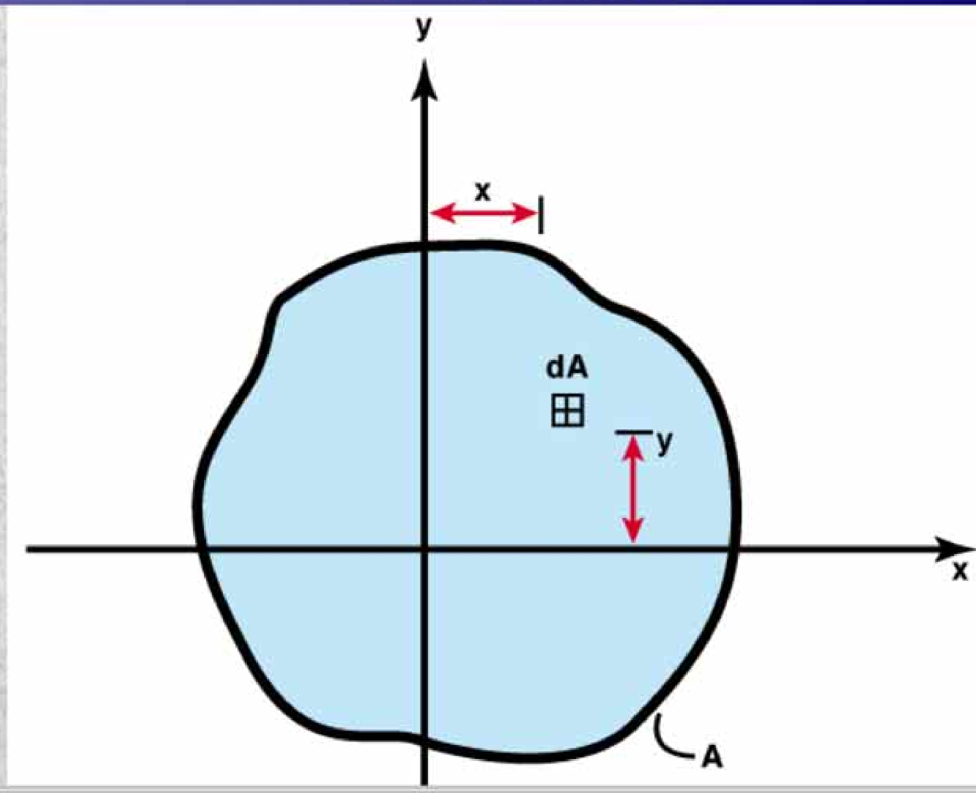
First and Second Moments:

$$A = \int_A dA$$

$$= \text{area}$$

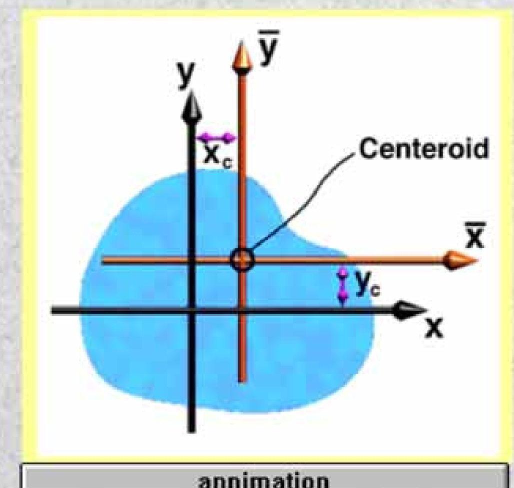


Geometric Properties of Plane Cross Sections



Geometric Properties of Plane Cross Sections

First Moments:



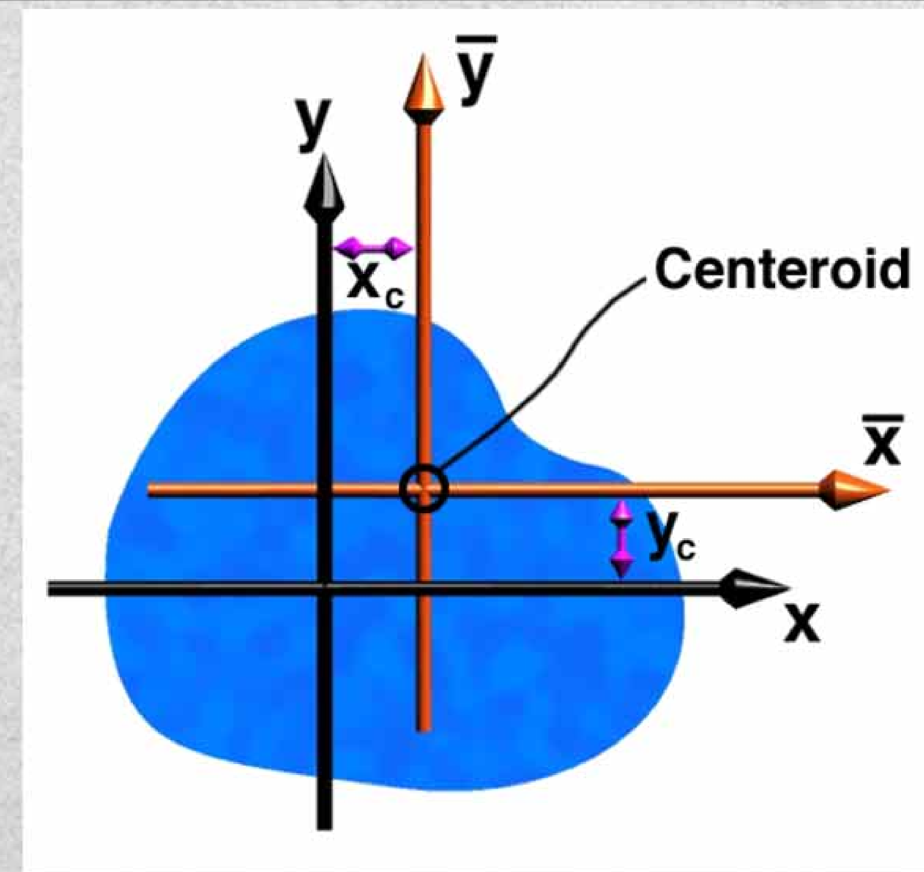
$$\begin{Bmatrix} S_x \\ S_y \end{Bmatrix} = \int_A \begin{Bmatrix} y \\ x \end{Bmatrix} dA$$

Coordinates of Centroid

$$\begin{Bmatrix} y_c \\ x_c \end{Bmatrix} = \frac{1}{A} \begin{Bmatrix} S_x \\ S_y \end{Bmatrix}$$

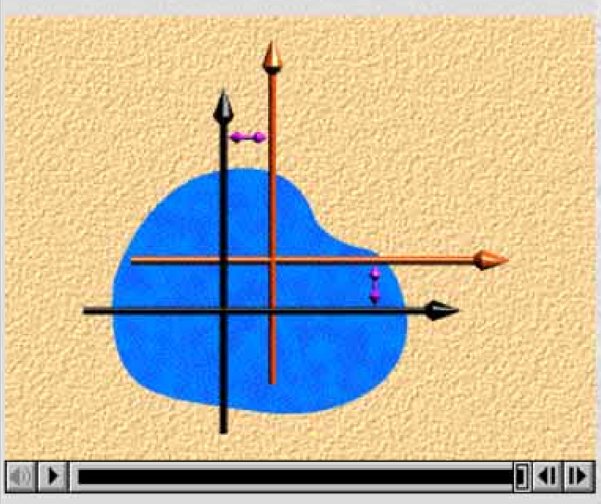
For Centroidal Axes $\bar{x}$, $\bar{y}$

$$S_x = S_y = 0$$



Geometric Properties of Plane Cross Sections

First Moments:



$$\begin{Bmatrix} S_x \\ S_y \end{Bmatrix} = \int_A \begin{Bmatrix} y \\ x \end{Bmatrix} dA$$

Coordinates of Centroid

$$\begin{Bmatrix} y_c \\ x_c \end{Bmatrix} = \frac{1}{A} \begin{Bmatrix} S_x \\ S_y \end{Bmatrix}$$

For Centroidal Axes $\bar{x}$, $\bar{y}$

$$S_x = S_y = 0$$

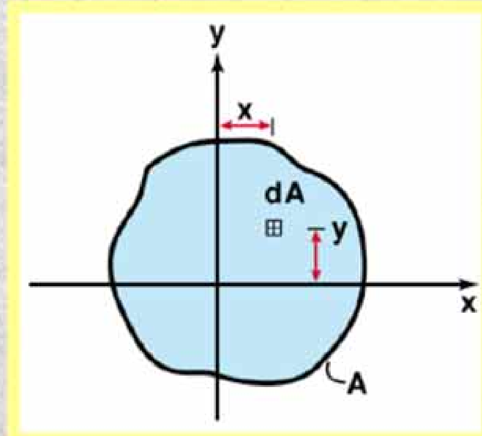
Geometric Properties of Plane Cross Sections

Second Moments (moments and product of Inertia):

$$\begin{Bmatrix} I_x \\ I_y \\ I_{xy} \end{Bmatrix} = \int_A \begin{Bmatrix} y^2 \\ x^2 \\ xy \end{Bmatrix} dA$$

I_p = polar second moment (moment of inertia)

$= I_x + I_y$

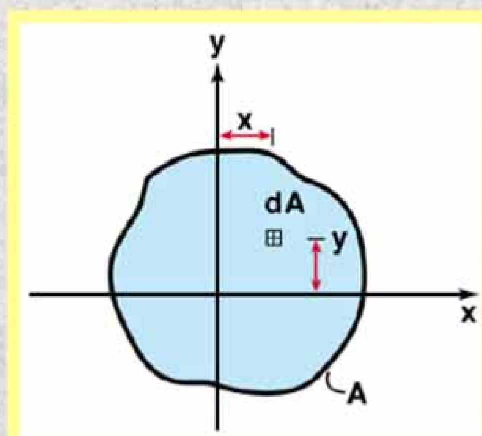


Geometric Properties of Plane Cross Sections

Radii of Gyration:

$$\begin{Bmatrix} r_x^2 \\ r_y^2 \end{Bmatrix} = \frac{1}{A} \begin{Bmatrix} I_x \\ I_y \end{Bmatrix}$$

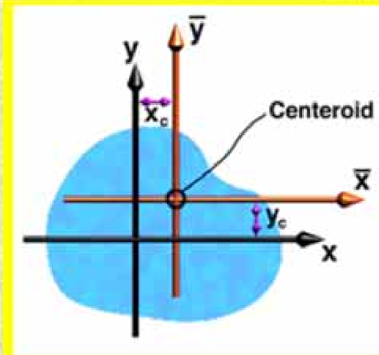
r_x, r_y are radii of gyration with respect to x and y axes



Geometric Properties of Plane Cross Sections

Effect of Translation of Coordinates

Translation of Coordinates



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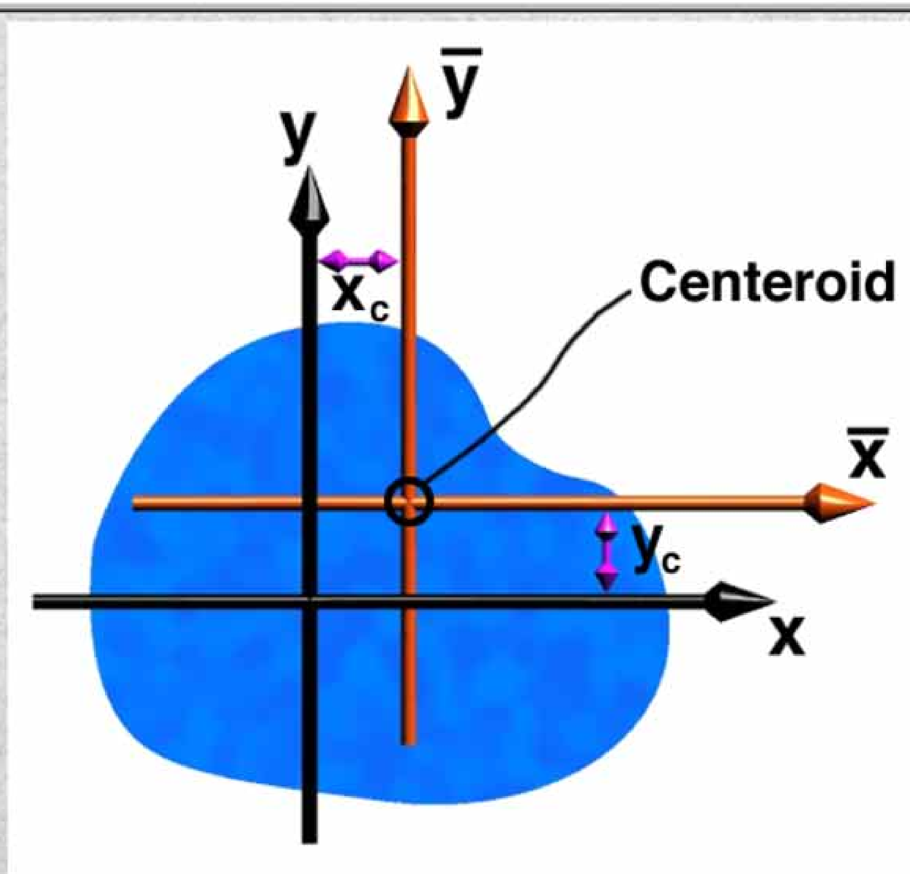
$\bar{x}$, $\bar{y}$ are centroidal coordinates

$$\left\{\begin{matrix} I_x \\ I_y \\ I_{xy} \end{matrix}\right\} = \left\{\begin{matrix} I_{\bar{x}} \\ I_{\bar{y}} \\ I_{\bar{x}\bar{y}} \end{matrix}\right\} + A\left\{\begin{matrix} y_c^2 \\ x_c^2 \\ x_c y_c \end{matrix}\right\}$$

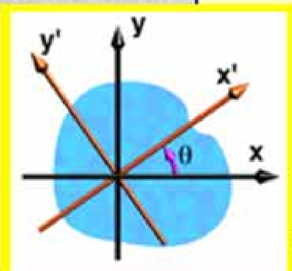


Geometric Properties of Plane Cross Sections

Rotation of Coordinates:

$$\begin{Bmatrix} I_{x'} \\ I_{y'} \\ I_{x'y'} \end{Bmatrix} = \begin{vmatrix} \cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{vmatrix} \begin{Bmatrix} I_x \\ I_y \\ I_{xy} \end{Bmatrix}$$

$$= \begin{vmatrix} \frac{1}{2}\left(1+\cos 2\theta\right) & \frac{1}{2}\left(1-\cos 2\theta\right) & -\sin 2\theta \\ \frac{1}{2}\left(1-\cos 2\theta\right) & \frac{1}{2}\left(1+\cos 2\theta\right) & \sin 2\theta \\ \frac{1}{2}\sin 2\theta & -\frac{1}{2}\sin 2\theta & \cos 2\theta \end{vmatrix} \begin{Bmatrix} I_x \\ I_y \\ I_{xy} \end{Bmatrix}$$



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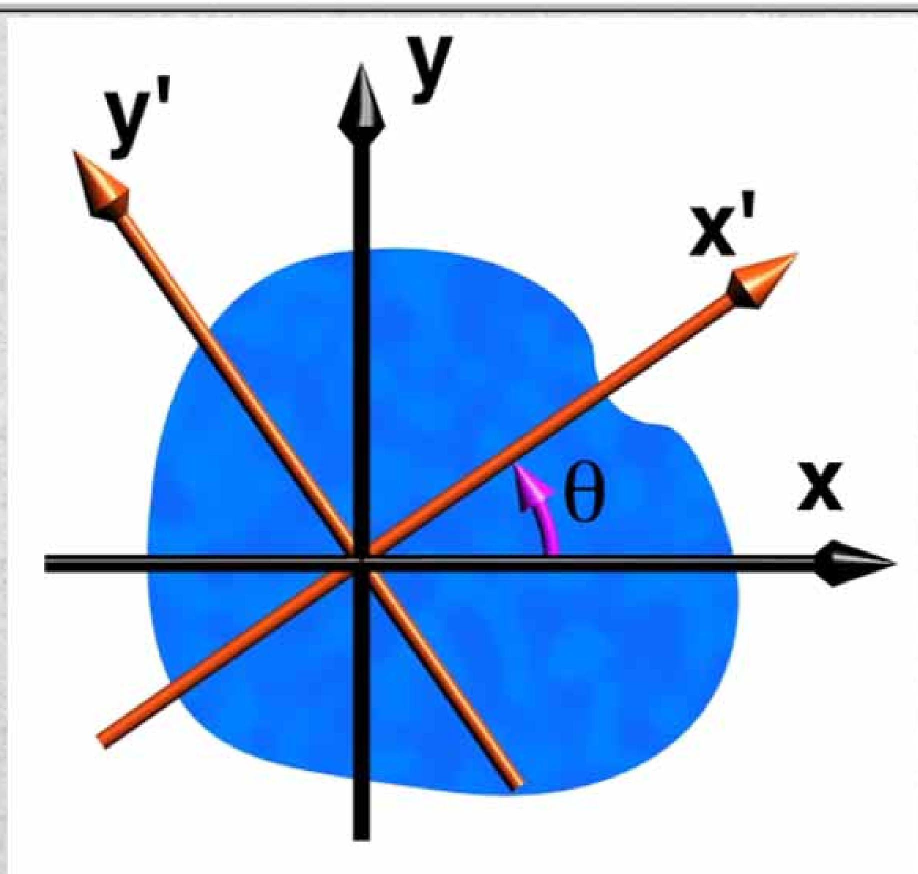


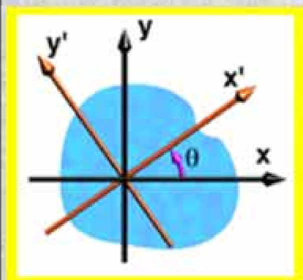
Geometric Properties of Plane Cross Sections

Principal Axes:

$I_{x'}$, $I_{y'}$ are maximum and minimum second moments

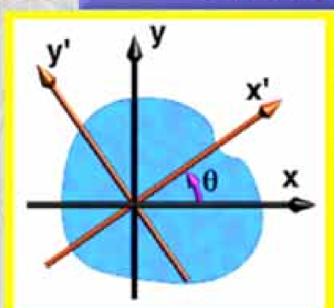
$$I_{x'y'} = 0$$

$$\tan 2\theta = \frac{-2\, I_{xy}}{I_x - I_y}$$



animation

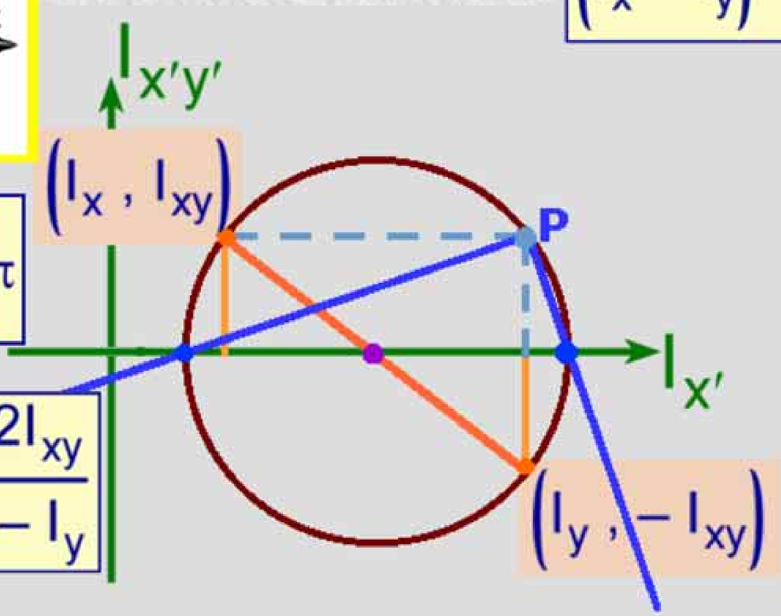
Mohr's Circle Representation of Moments and Product of Inertia



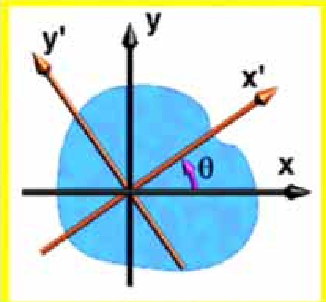
$$I_{xy} > 0$$
$$\left(I_x - I_y\right) < 0$$

$$\pi < 2\theta < \frac{3}{2}\pi$$

$$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$$



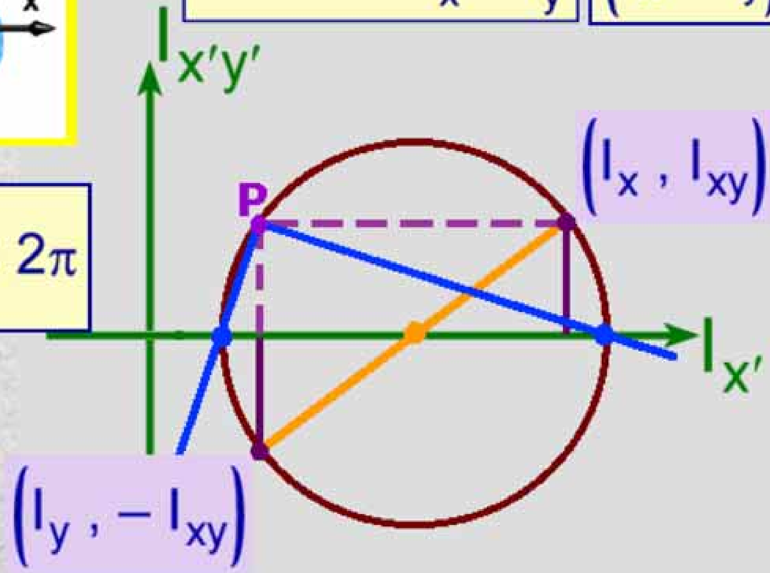
Mohr's Circle Representation of Moments and Product of Inertia



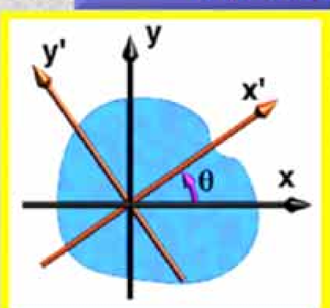
$$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$$

$$I_{xy} > 0$$
$$\left(I_x - I_y\right) > 0$$

$$\frac{3}{2}\pi < 2\theta < 2\pi$$



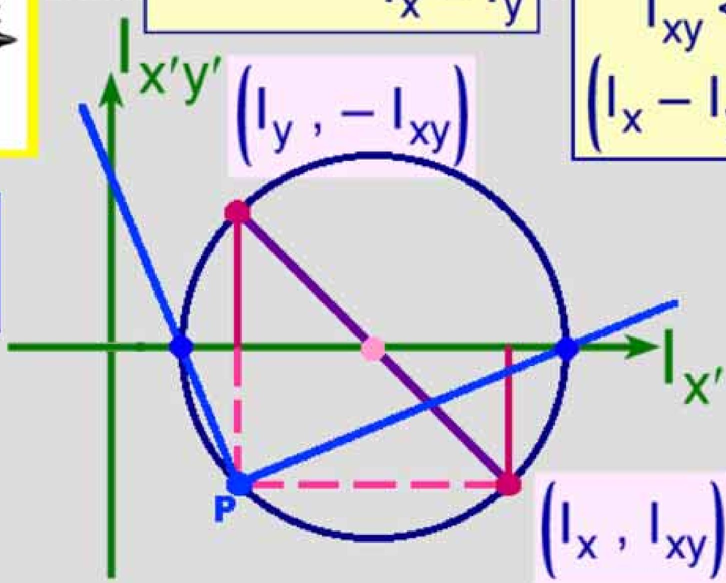
Mohr's Circle Representation of Moments and Product of Inertia

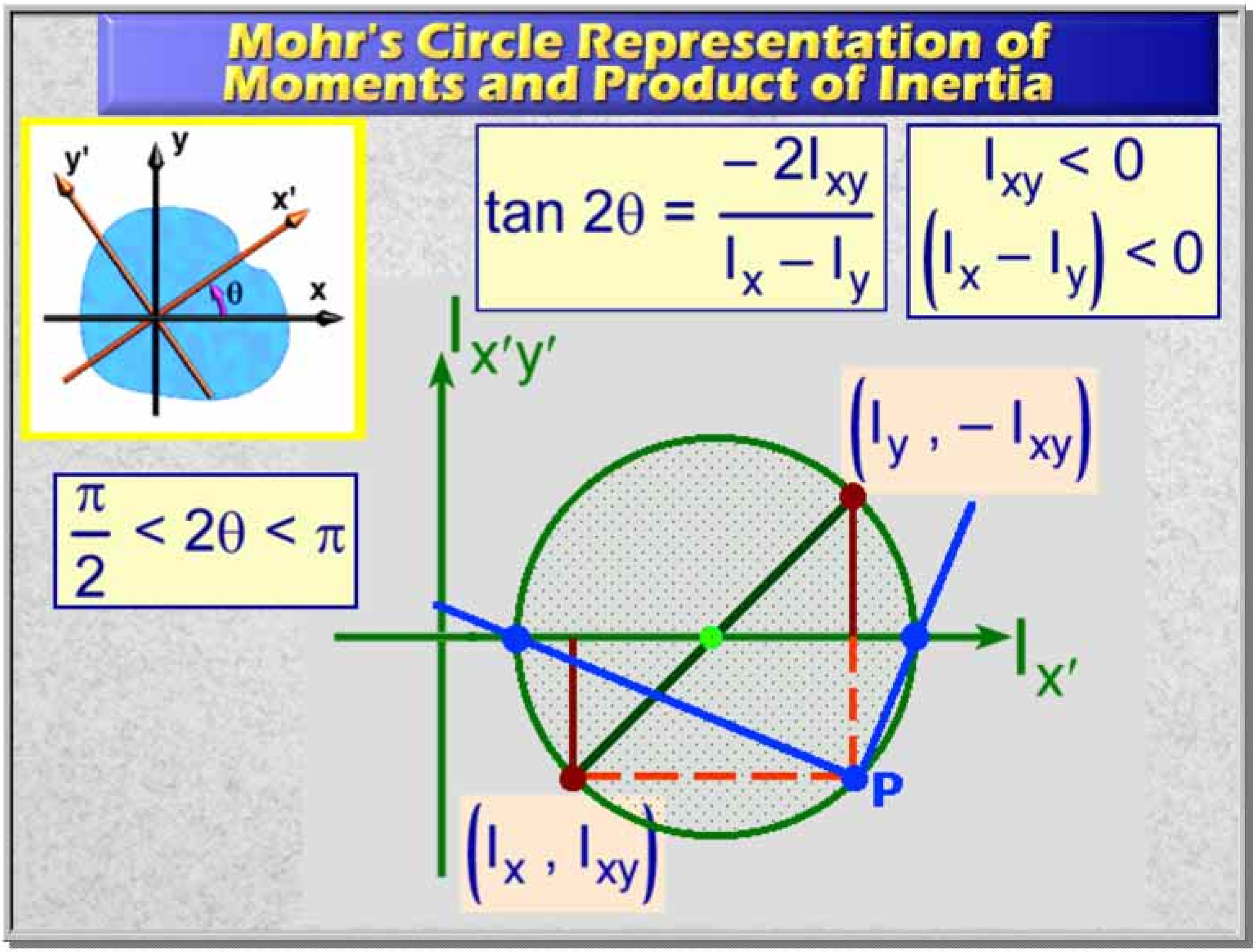


$$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$$

$$I_{xy} < 0$$
$$\left(I_x - I_y\right) > 0$$

$$0 < 2\theta < \frac{\pi}{2}$$




Mohr's Circle Representation of Moments and Product of Inertia
y'
y
x'
θ
x
$\tan 2\theta = \dfrac{-2I_{xy}}{I_x - I_y}$
$I_{xy} < 0$
$\left(I_x - I_y\right) < 0$
$\dfrac{\pi}{2} < 2\theta < \pi$
$I_{x'y'}$
$\left(I_y\,,\,-I_{xy}\right)$
$I_{x'}$
P
$\left(I_x\,,\,I_{xy}\right)$